O'NG TOMONDAN TASODIFIY SENZURLANISHNING UMUMIY MODELIDA TAQSIMOT FUNKSIYASINING YADROVIY BAHOLARI VA ULARNI SONLI SOLISHTIRISH

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Annotatsiya. *Ushbu maqolada o'ng tomondan tasodifiy senzurlanish modelni qarashimiz sababi, ushbu modelni metodik jihatdan tushuntirish juda qulaydir. Senzurlanishning bir nechta turlari mavjud (o'ngdan, chapdan, har ikkala tomondan, tavakkallik risklar bilan aralashtirilgan va boshqalar). Bunday modellar meditsina, biologiya, sug'urta ishi, demografiya va sanoat tajribalarida, umuman olganda amaliyot bilan bog'liq bo'lgan ko'plab sohalarda uchrab turadi. Oldindan silliqlangan darajadagi tavakkalliklar nisbati funksiyasi asosida o'ng tomondan tasodifiy senzurlanish modelida taqsimot funksiyasini noparametrik baholash muhokama qilinadi.*

Kalit so'zlar: *Asimptotik ifodalash, asimptotik normallik, kuchli asoslilik, proporsional intensivliklar modeli, tasodifiy senzurlanish, darajadagi tavakkalliklar nisbati bahosi, oldindan silliqlangan darajadagi tavakkalliklar nisbati bahosi, senzura qilingan ma'lumotlar, taqsimot funktsiyasi, integral intensivlik funksiyasi.*

ЯДЕРНЫЕ ОЦЕНКИ ФУНКЦИИ РАСПРЕДЕЛЕНИЯ И ИХ ЧИСЛЕННОЕ СРАВНЕНИЕ В ОБЩЕЙ МОДЕЛИ СЛУЧАЙНОГО ЦЕНЗИРОВАНИЯ СПРАВА

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Аннотация. *В статье обсуждается оценки функции распределения данных о продолжительности жизни, подвергнутых случайному цензурированию с правой стороны. В этом исследовании изучаем оценку функции распределения, используя понятие предварительно сглаженной оценки функции относительного риска. Характеристики оценки исследуются с использованием подходов цифрового моделирования. Показано, что предварительно сглаженная оценка степени относительного риска лучше соответствующей множительной оценки. Существует несколько схем цензурирования (справа, слева, с обеих сторон, в сочетании с конкурирующими рисками и другими). Однако в статистической литературе широко распространена случайное цензурирование с правой стороны, поскольку она легко описывается с методологической точки зрения. Здесь рассматривается этот вид цензурирования, чтобы сравнить наши результаты с другими.*

Ключевые слова: *Асимптотическое представление, асимптотическая* *нормальность, строгая состеятельность, сглаженные, модель пропорциональных интенсивностей, случайное цензурирование, функция относительного риска, предварительно сглаженная степень относительного риска, функция распределения, интегральная функция опасности интенсивности.*

IN THE GENERAL MODEL OF RANDOM CENSORSHIP FROM THE RIGHT, THE NUCLEAR ESTIMATES OF THE DISTRIBUTION FUNCTION AND THEIR NUMERICAL COMPARISON

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Annotation. *The article discusses the estimation of the distribution function of lifetime data subjected to right random censoring. We explore estimate of the distribution function in this study using the notion of presmoothed estimation of the relative-risk function. The characteristics of the estimator are investigated using numerical modeling approaches. We demonstrate that the presmoothed relative-risk power estimator outperforms the related product-limit estimator. There are several schemas of censoring (from the right, left, both sides, mixed with competing risks and others). However, in statistical literature right random censoring is wide spread, in so far as it was easily described from the methodological point of view. Here we consider also this kind of censorship in order to comparing our results with others.*

Keywords: *Asymptotic representation, asymptotic normality, strong consistency, proportional hazard model, random censorship, presmoothed relative-risk power, censored data, distribution function, cumulative hazard function.*

I. KIRISH

Dastlab O'ng tomondan tasodifiy senzurlanish modeli to'liq bo'lmagan tanlamalarning keng tarqalgan namoyondasi bo'lib, u tasodifiy senzurlanish modellari umr davomiyligini aniqlash tajribalarida, tibbiy-biologik, muxandislik tajribalarda va shu kabi boshqa tajribalarda uchrab turadi. **To'liq bo'lmagan tanlamalar A.A.Abdushukurovning [1] monografiyasida batafsil yoritilgan. Unda to'liq bo'lmagan tanlanmalarning xilma-xil umumlashgan modellarida taqsimot funksiyasi, uning bir qator funksionallarini baholash va olingan baholarni asimptotik xossalarini tadqiq qilish masalalari chuqur taxlil qilib o'rganilgandir.**

II.METODOLOGIYA

To‘liq bo‘lmagan tanlanmalarning o‘ng tomondan tasodifiy senzurlangan modeli quyidagicha aniqlanadi. Yagona ehtimollar fazosida taqsimot funksiyalari $F(x)=P(\xi \le x)$ va $G(y)=P(\zeta \le y),\ x,y \in \text{Ў}$, bo‘lgan ξ va ζ tasodifiy miqdorlarni qaraymiz. (ξ,ζ) juftlikni n ta bo‘g‘liq bo‘lmagan tajribalardagi amaliy qiymatlari mos ravishda $(X_1,Y_1),(X_2,Y_2),...,(X_n,Y_n)$ lar bo‘lsin. Shunday tajribalarni qaraymizki, har bir i – tajribada (ξ_i,δ_i) juftlik kuzatiladi va natijada hajmi n ga teng bo‘lgan $\text{J}^{(n)}=\{(\xi_1,\delta_1),(\xi_2,\delta_2),...,(\xi_n,\delta_n)\}$ - statistik tanlanmaga ega bo‘lamiz. Bu yerda $\xi_i=\min(X_i,Y_i)$ va $\delta_i=I(X_i\le Y_i)$ va $I(A)$ esa indikator. Demak, agar $\delta_i=1$ bo‘lsa, $\xi_i=X_i$ va aks holda $\delta_i=0$ da $\xi_i=Y_i$. Demak, barcha kuzatilayotgan X_i lar soni $\nu_n=\delta_1+...+\delta_n$ ga va Y_i lar soni esa $n-\nu_n$ ga teng. Bizning X_i tasodifiy miqdorlar (demak, F -taqsimotlar) qiziqtirayotgan bo‘lib, Y_i lar (demak, G -taqsimotlar esa) halaqit beruvchi bo‘lsin. Asosiy masala $\text{J}^{(n)}$ tanlanma bo‘yicha F taqsimotni yoki uning biror funksionalini baholashdan iborat bo‘lsin. Masalan, $F(x)$ uchun $\text{J}^{(n)}$ tanlanma bo‘yicha tabiiy statistika:

$$\hat{F}_n(x)=\frac{1}{\nu_n}\sum_{i=1}^{n}\delta_i\cdot I(\xi_i\le x)$$

hattoki asosli baho ham bo‘lmas ekan. Haqiqatan ham, $n\to\infty$ da

$$\hat{F}_n(x)\underset{n\to\infty}{\overset{P}{\to}}\hat{F}(x)=\frac{\int_{-\infty}^{x}(1-G(t))dF(t)}{\int_{-\infty}^{\infty}(1-G(t))dF(t)},$$

yaqinlashish o‘rinli ekan ([2] ga qarang). Ko‘rinib turibdiki, ixtiyoriy x da

$$\hat{F}(x)\ne F(x),\ x\in \text{Ў}.$$

Demak, $\hat{F}_n(x)$ statistika $F(x)$ uchun baho bo‘la olmas ekan. Biz empirik baho kabi siljimaganlik va asoslilik xossalariga ega bo‘lgan bahoni quramiz va bu uchun ba’zi qo‘shimcha shartlar bajarilishini talab qilamiz. Faraz qilaylik, $G(x)$

taqsimot funksiyasi ***ma'lum bo'lsin***. Ushbu statistik modelda ξ_i larning taqsimot funksiyasi

$$H(x)=P(\xi_i\le x)=P\left(\min(X_i,Y_i)\le x\right)=1-P\left(\min(X_i,Y_i)>x\right)=$$
$$=1-P(X_i>x,Y_i>x)=1-P(X_i>x)P(Y_i>x)=1-\left(1-F(x)\right)\left(1-G(x)\right),\ x\in \text{Ў}$$

ga teng bo'ladi.

Quyidagi subtaqsimotlarni kiritamiz:

$$H^{(0)}(x)=P(\xi_i\le x,\delta_i=0)=P(Y_i\le x,Y_i<X_i)=\int_{-\infty}^{x}\left(1-F(t)\right)dG(t),\ x\in \text{Ў},$$

$$H^{(1)}(x)=P(\xi_i\le x,\delta_i=1)=P(X_i\le x,Y_i\le X_i)=\int_{-\infty}^{x}\left(1-G(t)\right)dF(t),\ x\in \text{Ў}.$$

Tabiiyki, barcha $x\in \text{Ў}$ lar uchun $H^{(0)}(x)+H^{(1)}(x)=H(x)$ hamda $p=P(\xi\le\zeta)=P(\delta_i=1)=H^{(1)}(+\infty)=\lim\limits_{x\to+\infty}H^{(1)}(x)$ va $1-p=P(\xi>\zeta)=P(\delta_i=0)=H^{(0)}(\infty)=\lim\limits_{x\to+\infty}H^{(0)}(x)$. Demak, barcha x larda qaralayotgan modelda F noma'lum bo'lgani uchun $H^{(0)},H^{(1)}$ va H lar ham noma'lumdir. Bu funksiyalar uchun $J^{(n)}$ tanlanma bo'yicha tabiiy empirik baholar mos ravishda $H_n^{(0)},H_n^{(1)}$ va H_n lar bo'lib, bunda

$$H_n^{(0)}(x)=\frac{1}{n}\sum_{i=1}^{n}(1-\delta_i)\cdot I(\xi_i\le x),$$
$$H_n^{(1)}(x)=\frac{1}{n}\sum_{i=1}^{n}\delta_i\cdot I(\xi_i\le x),\tag{1}$$
$$H_n(x)=H_n^{(0)}(x)+H_n^{(1)}(x)=\frac{1}{n}\sum_{i=1}^{n}I(\xi_i\le x),$$

(1) empirik baholar tabiiy siljimaganlik va tekis kuchli asoslilik xossalariga ega. Endi $G(x)$ taqsimot funksiya ma'lum bo'lgan holda

$$\hat{F}_n(x)=\hat{F}_{nG}(x)=\int_{-\infty}^{x}\frac{dH_n^{(1)}(t)}{1-G(t)}=\frac{1}{n}\sum_{i=1}^{n}\frac{\delta_i\cdot I(\xi_i\le x)}{1-G(\xi_i)},\quad x\le\tau_G=\{x:G(x)=1\}$$

statistikani $F(x)$ uchun baho sifatida tanlash mumkin. U quyidagi xossalarga ega:

1) Barcha $x < \tau_G$ lar uchun

$$M\hat{F}_{nG}(x) = F(x) \quad (\textbf{\textit{siljimaganlik}}):$$

$$M\hat{F}_{nG}(x) = M\left\{\frac{\delta_i \cdot I(\xi_i \le x)}{1 - G(\xi_i)}\right\} = \int_{-\infty}^{x} \frac{dH^{(1)}(t)}{1 - G(t)} = F(x),$$

2) har bir $x < \tau_G$ va $n \to \infty$ da, Chebishev tengsizligiga asosan,

$$\hat{F}_n(x) \underset{n\to\infty}{\overset{P}{\to}} F(x) \quad (\textbf{\textit{nuqtada asosli}}),$$

3) $P\left(\sup_{x<\tau_G}\left|\hat{F}_n(x) - F(x)\right| \underset{n\to\infty}{\to} 0\right) = 1$ (***tekis kuchli asosli***), (***Glivenko-Kontelli teoremasi***).

4) Har bir $x < \tau_G$ va $n \to \infty$ da ***asimptotik normal***

$$\sqrt{n}\left(\hat{F}_n(x) - F(x)\right) \underset{n\to\infty}{\overset{d}{\to}} \Gamma\left(0, \sigma_G^2(x)\right),$$

bu yerda $\sigma_G^2(x) = \int_{-\infty}^{x} \frac{dF(t)}{1 - G(t)} - F^2(x)$.

Yuqoridagi barcha hisoblardan ko‘rinadiki, $(-\infty; \tau_G]$ oraliqda $\hat{F}_n(x)$ statistika $F(x)$ uchun $G(x)$ ma’lum bo‘lgan holda yaxshi baho bo‘lar ekan.

Faraz qilaylik, $X_1, X_2, \ldots$ va $Y_1, Y_2, \ldots$ lar bog‘liq bo‘lmagan va mos ravishda uzluksiz F va G taqsimot funksiyalariga ega tasodifiy miqdorlar ketma-ketliklari bo‘lsin. Kuzatilayotgan tanlanma:

$$J^{(n)} = \left\{(Z_j, \delta_j), 1 \le j \le n\right\}$$

dan iborat bo‘lsin. Bu yerda n-tanlanma hajmi, $Z_j = min(X_j; Y_j)$ va $\delta_j = I(X_j \le Y_j) = I(Z_j = X_j)$, $I(A)-$ A hodisaning indikatori. $I(A) = 1$, agar A ro‘y bersa, $I(A) = 0$ aks holda. Tanlanmaning tuzilishidan tushunalliki:

- Agar $X_j \le Y_j$ bo‘lsa, $Z_j = min(X_j; Y_j) = X_j$ ga teng bo‘lganda $\delta_j = 1$ ga teng bo‘ladi va bu holatda biz X ni kuzata olamiz;

- Agar $Y_j \le X_j$ bo‘lsa, u holda $Z_j = min\left(X_j;Y_j\right) = Y_j$ ga teng bo‘lganda $\delta_j = 0$ bo‘ladi bu esa senzurlanishli holat bo‘ladi.

Shu sababli manashu kuzatilayotgan $J^{(n)}$ tanlanmada bizni qiziqtirayotgan X_j larning soni tasodifiy bo‘ladi ya’ni $\nu_n = \delta_1 + \delta_2 + ... + \delta_n$ tasodifiy sondagi 1 larning yig‘indisi bu ***Binomial*** $B_i\left(n; p\right)$ taqsimotga ega bo‘lgan tasodofiy miqdor.

Asosiy masala Y_i larning $G\left(t\right)$ ***taqsimot funksiyasi noma’lum bo‘lgan holda*** X_i larning taqsimot funksiyasi $F\left(t\right)$ ni $\left(Z_j;\delta_j\right)$ juftliklardan iborat bo‘lgan $J^{(n)}$ tanlanma bo‘yicha baholashdan iborat.

$F\left(t\right)$ uchun dastlab noparametrik (**Product–Limit** (**PL**)) ko‘paytma ko‘rinishidagi baho Kaplan va Mayer tomonidan [3] da taklif etilgan.

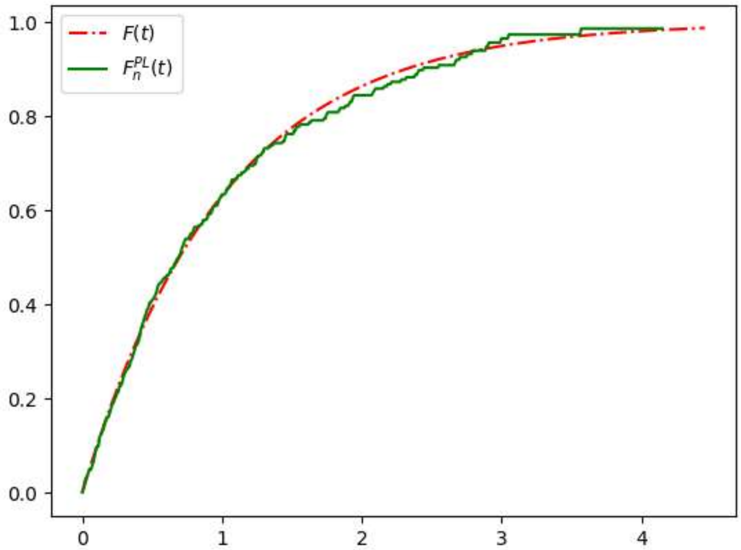
U quyidagicha aniqlanadi:

$$F_n^{PL}\left(t\right) = \begin{cases} 1 - \prod\limits_{\left\{j: Z_{(j)} \le t\right\}} \left(1 - \dfrac{\delta_{(j)}}{n - j + 1}\right), & t \le Z_{(n)}, \\ 1, & t > Z_{(n)}, \quad va \quad \delta_{(n)} = 1, \\ aniqlanmagan, & t > Z_{(n)}, \quad va \quad \delta_{(n)} = 0, \end{cases} \tag{2}$$

bu yerda $Z_{(1)} \le Z_{(2)} \le ... \le Z_{(n)}$ qator $\left\{Z_j, j = 1,...,n\right\}$ lardan tuzilgan tartiblangan variatsion qator bo‘lib $\left\{\delta_{(j)}, j = 1,...,n\right\}$, esa ularga mos keladigan indikatordir. (2) baho ko‘pgina statistik masalalarda mutaxasislar tomonidan tadqiq qilib qullanib kelinmoqda. PL-bahoning boshqa ko‘rinishlari ham mavjud. Biroq, bu bahoning kamchiliklaridan biri Z_j eng katta qiymatida $Z_{(n)}$ da senzurlanishga bog‘liq bo‘lib qoladi ((2) ga qarang). PL-bahoning turli xossalarini o‘rganish va ularni statistik masalalarda qo‘llash borasida juda ko‘plab adabiyotlar bor ([4] larga qarang).

Endi yuqoridagi (2) bahoni qay darajada ekanligini o‘rganish maqsadida kompyuter modellashtirish yordamida ushbu model bo‘yicha tanlanma beradigan

Pythonda dastur tuzib, bahoni chizishda bosh to‘plam taqsimoti sifatida $F(t)=1-e^{-t}$, $t\geq 0$, -eksponensial taqsimotni olib, undan $n=500$ hajmli senzurlanish darajasi 30% (150 ta) bo‘lgan sun’iy tanlanma olamiz va u yordamida quyidagi chizmani chizib ko‘ramiz (1-rasm).



1-rasm. $F_n^{PL}(t)$ – Kaplan-Mayer bahosi

Ammo (2) $F(t)$ uchun yagona baho emas, yana bir noparametrik darajali ko‘rinishda bo‘lib, uni Abdushukurov [5] tomonidan “Darajadagi Tavakkalliklar Nisbati bahosi” (**Relative-Risk Power** (**RR**)) deb nomlangan bahoni taklif etilgan bo‘lib, uning ko‘rinishi esa quyidagidek aniqlanadi:

$$F_n^{RR}(t)=1-\left(1-H_n(t)\right)^{R_n(t)}=\begin{cases}0, & t<Z_{(1)},\\ 1-\left(\dfrac{n-j}{n}\right)^{R_n(t)}, & Z_{(j)}\leq t<Z_{(j+1)}, 1\leq j\leq n-1,\\ 1, & t\geq Z_{(n)},\end{cases} \quad (3)$$

bu yerda $H_n(t)=\frac{1}{n}\sum_{j=1}^{n}I\left(Z_j\leq t\right)$, $t\in \mathbb{R}^1\equiv(-\infty;+\infty)$, $H_n(t)$ baho $H(t)$ uchun empirik bahodir. Bu yerda

$$H(t)=P\left(Z_j\leq t\right)=1-P\left(Z_j\geq t\right)=1-P\left(X_j\geq t\right)P\left(Y_j\geq t\right)=1-\left(1-F(t)\right)\left(1-G(t)\right)$$

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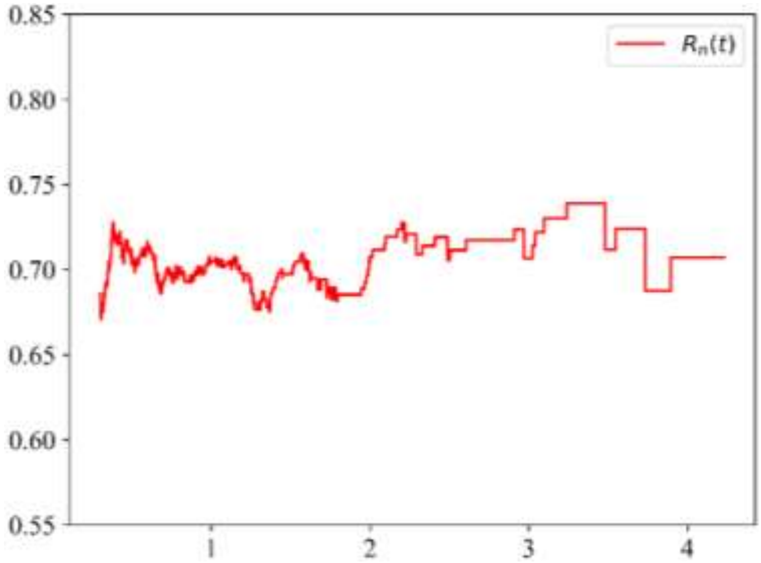
minimumlarning taqsimot funksiyasi, $R_n(t) = \dfrac{\Lambda_{1n}(t)}{\Lambda_n(t)}$ va $R(t) = \dfrac{\Lambda_1(t)}{\Lambda(t)}$, $t \in \text{Ў}$.

$R_n(t)$ baho intensivliklar nisbati funksiyasi $R(t)$ ning bahosi. Endi biz ushbu $R(t) = \dfrac{\Lambda_1(t)}{\Lambda(t)}$, $t \in \text{Ў}$ intensivliklar nisbati funksiyasini qaraymiz. Bu yerdagi *H, G* va *F* taqsimot funksiyalariga mos kelgan Λ, Λ_0 va Λ_1 kumulyativ-integral intensivliklar funksiyalari quyidagicha aniqlanadi:

$$\begin{aligned}
\Lambda(t) &= \int_{-\infty}^{t} \frac{dH(u)}{1-H(u)} = \Lambda_0(t) + \Lambda_1(t),\\
\Lambda_0(t) &= \int_{-\infty}^{t} \frac{dG(u)}{1-G(u)} = \int_{-\infty}^{t} \frac{dH_0(u)}{1-H(u)} = -\ln\left(1-G(t)\right),\\
\Lambda_1(t) &= \int_{-\infty}^{t} \frac{dF(u)}{1-F(u)} = \int_{-\infty}^{t} \frac{dH_1(u)}{1-H(u)} = -\ln\left(1-F(t)\right).
\end{aligned} \tag{4}$$

(4) formulalardan ko'rinadiki, $R(t) = \dfrac{\Lambda_1(t)}{\Lambda(t)} = \dfrac{\Lambda_1(t)}{\Lambda_0(t)+\Lambda_1(t)}$, $t \in (-\infty;+\infty)$ funksiya $[0;1]$ da chegaralangan.

Endi yuqoridagi (3) bahonining darajasidagi $R_n(t)$ bahoni qay darajada ekanligini o'rganish maqsadida kompyuter modellashtirish yordamida ushbu model bo'yicha tanlanma beradigan Pythonda dastur tuzib, bahoni chizib ko'ramiz. Bahoni chizishda bosh to'plam taqsimoti sifatida $F(t) = 1 - e^{-t}$, $t \geq 0$, -eksponensial taqsimotni olib, undan $n = 500$ hajmli senzurlanish darajasi 30% (150 ta) bo'lgan sun'iy tanlanma olamiz va u yordamida quyidagi chizmani chizamiz (2-rasm).



2-rasm. $R_n(t)$ – intensivliklar nisbati funksiyasi bahosi

$H(t)$ – taqsimot funksiyasi esa quyidagi subtaqsimotlar yig'indisidan iborat bo'lib,

$$H(t) = H_0(t) + H_1(t), t \in \mathbb{R}^1,$$

bu yerda

$$H_0(t) = P\left(Z_j \le t, \delta_j = 0\right) = \int_{-\infty}^{t} \left(1 - F(u)\right) dG(u),$$

$$H_1(t) = P\left(Z_j \le t, \delta_j = 1\right) = \int_{-\infty}^{t} \left(1 - G(u)\right) dF(u),$$

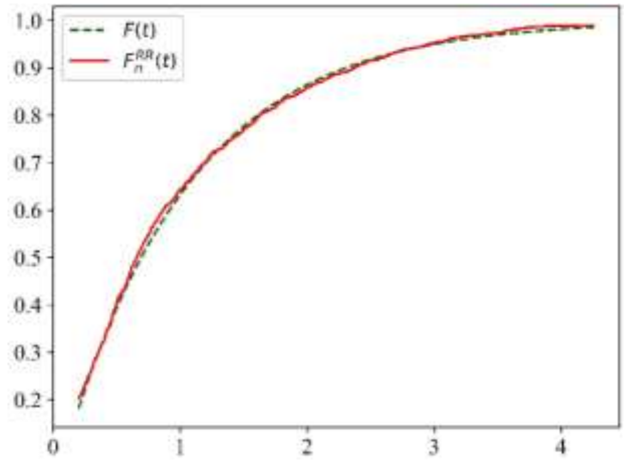
impirik statistikalar esa

$$\Lambda_n(t) = \int_{-\infty}^{t} \frac{dH_n(u)}{1 - H_n(u)} = \Lambda_{0n}(t) + \Lambda_{1n}(t),$$

ko'rinishida bo'lib, bunda

$$\Lambda_{kn}(t) = \int_{-\infty}^{t} \frac{dH_{kn}(u)}{1 - H_n(u)}, \qquad H_{kn}(t) = \frac{1}{n}\sum_{j=1}^{n} I\left(Z_j \le t, \quad \delta_j = k\right), \ k = 0, 1$$

Endi yuqoridagi (3) bahoni qay darajada ekanligini o'rganish maqsadida kompyuter modellashtirish yordamida ushbu model bo'yicha tanlanma beradigan Pythonda dastur tuzib, bahoni chizishda bosh to'plam taqsimoti sifatida $F(t) = 1 - e^{-t}$, $t \ge 0$, -eksponensial taqsimotni olib, undan hajmlari n=500 ga teng, senzurlanish darajasi 30 % (150 ta) boʻlgan sunʼiy tanlanma olamiz va u yordamida quyidagi grafiklarini chizamiz (3-rasmlar).



3-rasm. $F_n^{RR}(t)$ – Darajadagi tavakkalliklar nisbati bahosi

Ushbu 2-rasmlardan koʻrinib turibdiki, daraja koʻrinishidagi (3) baho (2) bahodan farqli ravishda senzurlanishga bogʻliqsiz holda $n \to \infty$ ga nazariy $F(t)$ taqsimotga tekis ravishda barcha $t \in \text{Ў}^1$ larda yaqinlashar ekan. $P\left(\lim\limits_{x\to\infty}\sup\limits_{t\in \text{Ў}}\left(F_n^{RR}(t) - F(t)\right) = 0\right) = 1$, yaʼni Glivenko-Kontellining teoremasi anologi oʻrinli ekan. [6] maqolalarda (2) va (3) baholar asimptotik ekvivalent, yaʼni $n \to \infty$ da ularning oʻxshash xossalarga ega ekanligi, ammo n ning kichik qiymatlari uchun (3) baho bir qator afzalliklarga ega ekanligi koʻrsatib oʻtilgan. Avvalambor (2) bahoning bir muhim kamchiligi uning variotsion qatorning oxirgi nuqtasida $\delta_{(n)} = 0$ boʻlgan holda aniqlanmaganligidir. Bu esa (2) bahoning senzurlanishga bogʻliq ekanligini koʻrsatadi. (3) bahoning afzalligi esa uning butun son oʻqida barcha t lar uchun aniqlanganligidadir. Haqiqatan ham, (3) bahoning nomaʼlum $G(t)$ uchun bahosi (3) formulaning oʻxshashi, yaʼni

$$G_n^{RR}(t) = 1 - \left(1 - H_n(t)\right)^{1-R_n(t)},\ t \in \text{Ў}, \qquad (5)$$

koʻrinishida boʻlib, oʻng tomondan senzurlanish modelida

$$1 - H(t) = \left(1 - G(t)\right)\left(1 - F(t)\right),\ t \in \text{Ў},$$

tenglikning empirik anologi boʻlmish

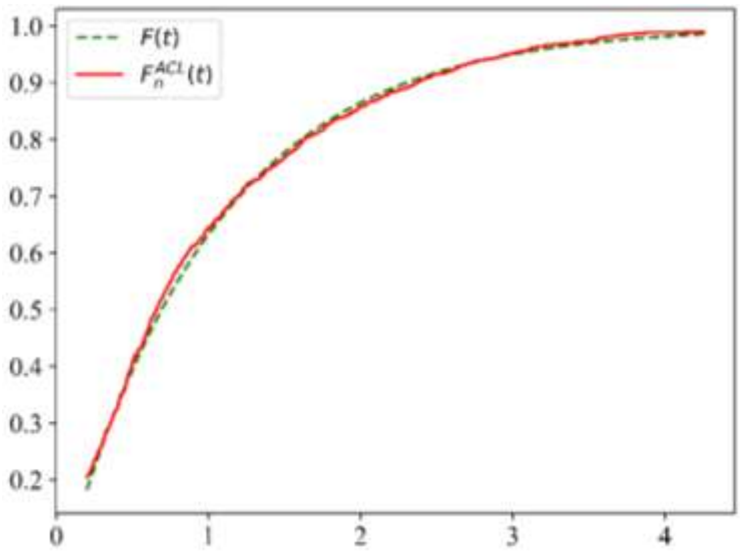
$$1-H_n(t)=\left(1-G_n(t)\right)\left(1-F_n(t)\right),\ t\in \text{Ў},$$

tenglikning o‘rinli ekanligidadir. Ko‘rish qiyin emaski, (3) va (5) baholar (**Abdushukurov-Cheng-Lin (ACL)** [7]) bahosining Proporsional Intensivlik Modelidagi (PIMdagi)

$$F_n^{ACL}(t)=1-\left[1-H_n(t)\right]^{p_n},\ G_n^{ACL}(t)=1-\left[1-H_n(t)\right]^{1-p_n},\ t\in \text{Ў}, \qquad (6)$$

o‘xshashlari kabi barcha yaxshi xossalarga egadir. PIMning asosiy xususiyati $\{Z_1,...,Z_n\}$ va $\{\delta_1,...,\delta_n\}$ larning bir biriga bog‘liq emasligidir. Eslatib o‘tamiz (6) baholar PIMda $1-G(t)=\left(1-F(t)\right)^{\beta}$, $t\in \text{Ў}$, $\beta>0$, tenglik o‘rinli bo‘lgan holda hisoblanar edi. Bu yerda $p=P\left(X_j\le Y_j\right)=\dfrac{1}{1+\beta}$ va $p_n=\dfrac{1}{n}\sum_{j=1}^{n}\delta_j$. PIMda $p=\dfrac{1}{1+\beta}$ bo‘lib, β - parameter esa senzurlanish darajasini aniqlaydi [8, 9].

Endi yuqoridagi ACL bahoni qay darajada ekanligini o‘rganish maqsadida kompyuter modellashtirish yordamida ushbu model bo‘yicha tanlanma beradigan Pythonda dastur tuzib, bahoni chizishda bosh to‘plam taqsimoti sifatida $F(t)=1-e^{-t}$, $t\ge 0$, -eksponensial taqsimotni olib, undan $n=500$ hajmli senzurlanish darajasi 30% (150 ta) bo‘lgan sun’iy tanlanma olamiz va u yordamida quyidagi chizmani chizib ko‘ramiz (4-rasm).



4-rasm. $F_n^{ACL}(t)$ – ACL bahosi

Biz endi (2) va (3) baholarni qisman silliqlash masalasini koʻrib chiqamiz. Hisoblardan koʻrinadiki, δ_j indikatorlarning Z_j larning berilgan qiymatlariga nisbatan regressiya funksiyasi quyidagi shartli matematik kutilmadan iborat boʻladi:

$$p(t) = P\left(\delta_j = 1 \mid Z_j = t\right) = M\left[\delta_j \mid Z_j = t\right], \ t \in Ў. \tag{7}$$

U holda $\Lambda_1(t)$ integral intensivlik funksiyasini quyidagidek ifodalash mumkin:

$$\Lambda_1(t) = \int_{-\infty}^{t} p(u)\, d\Lambda(u), \ t \in Ў. \tag{8}$$

Demak intensivliklar nisbati funksiyasi endi

$$R(t) = \frac{\Lambda_1(t)}{\Lambda(t)} = \frac{\int_{-\infty}^{t} p(u)\, d\Lambda(u)}{\Lambda(t)}, \ t \in Ў \tag{9}$$

koʻrinishida boʻladi. Endi (7) regrissiya funksiyasi uchun Nadaraya-Uotsonning quyidagi noparametrik regrissiya bahosini qullaymiz [10, 11]:

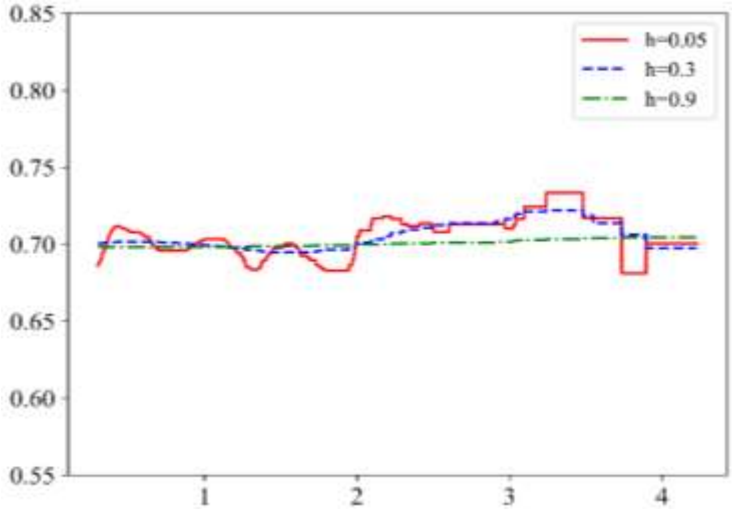
$$p_n(t) = \frac{\dfrac{1}{nh(n)} \sum_{j=1}^{n} \delta_j k\left(\dfrac{t - Z_j}{h(n)}\right)}{\dfrac{1}{nh(n)} \sum_{j=1}^{n} k\left(\dfrac{t - Z_j}{h(n)}\right)}, \tag{10}$$

bu yerda yadroviy bahodagi oyna kengligi $\left\{h(n) = n^{-\alpha}, 0 < \alpha < 1, n \in N\right\}$ ketma-ketligi $n \to \infty$ da $h(n) \downarrow 0$, hamda $k(t)$ esa zichlik (yadro) funksiyasidir. Natijada (8)–(9) formulalarga asosan $R(t)$ funksiyaning quyidagi bahosiga ega boʻlamiz [12]:

$$R_n^p(t) = \frac{\Lambda_{1n}^p(t)}{\Lambda_n(t)} = \frac{1}{\Lambda_n(t)} \cdot \int_{-\infty}^{t} p_n(u)\, d\Lambda_n(u), \ t \in Ў. \tag{11}$$

Endi yuqoridagi (11) intensivliklar nisbati funksiyasi $R_n^p(t)-$ bahoni qay darajada ekanligini oʻrganish maqsadida kompyuter modellashtirish yordamida ushbu model boʻyicha tanlanma beradigan Pythonda dastur tuzib, bahoni chizib koʻramiz. Bahoni chizishda bosh toʻplam taqsimoti sifatida $F(t) = 1 - e^{-t}, t \geq 0,$ -

eksponensial taqsimotni olib, undan $n = 500$ hajmli senzurlanish darajasi 30% (150 ta) bo‘lgan sun’iy tanlanma olamiz va u yordamida quyidagi chizmani chizamiz. Baho yadro funksiyasi va oyna kengligi parametrlariga bog‘liq bo‘lganligi uchun quyidagi chizma Normal yadro funksiyasi (empirik nuqtai nazardan baho eng yaxshi bo‘ladigan yadro) va h=0,05; 0,3; 0,9 parametrlar bilan chizildi (5-rasm).

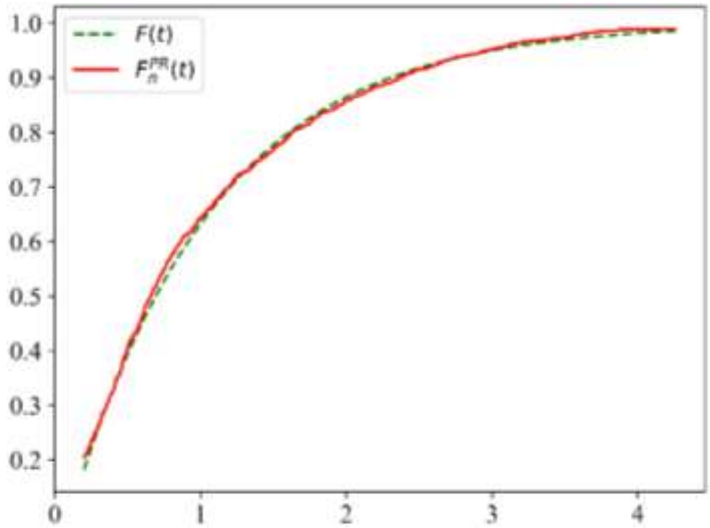


5-rasm. $R_n^p(t)$ – intensivliklar nisbati funksiyasi bahosi

Endi (11) bahoni (3) bahoning darajasidagi $R_n(t)$ ni o‘rniga qo‘ysak, Abdushukurov-Nurmukhamedova-Bozorovlar tomonidan taklif qilingan quyidagi yangi “Oldindan Silliqlangan Darajadagi Tavakkalliklar Nisbati bahosi” ga (**Presmoothed Relative-Risk Power** (**PRRP**)) ega bo‘lamiz [13]:

$$F_n^{PR}(t) = 1 - \left[1 - H_n(t)\right]^{R_n^p(t)} = \begin{cases} 0, & t < Z_{(1)}, \\ 1 - \left(\dfrac{n-j}{n}\right)^{R_n^p(t)}, & Z_{(j)} \le t < Z_{(j+1)}, 1 \le j \le n-1, \\ 1, & t \ge Z_{(n)}, \end{cases} \quad (12)$$

Endi yuqoridagi (12) bahoni qay darajada ekanligini o‘rganish maqsadida kompyuter modellashtirish yordamida ushbu model bo‘yicha tanlanma beradigan Pythonda dastur tuzib, bahoni chizishda bosh to‘plam taqsimoti sifatida $F(t) = 1 - e^{-t}$, $t \ge 0$, -eksponensial taqsimotni olib, undan $n = 500$ hajmli senzurlanish darajasi 30% (150 ta) bo‘lgan sun’iy tanlanma olamiz va u yordamida quyidagi bahoni chizib ko‘ramiz (6-rasm).



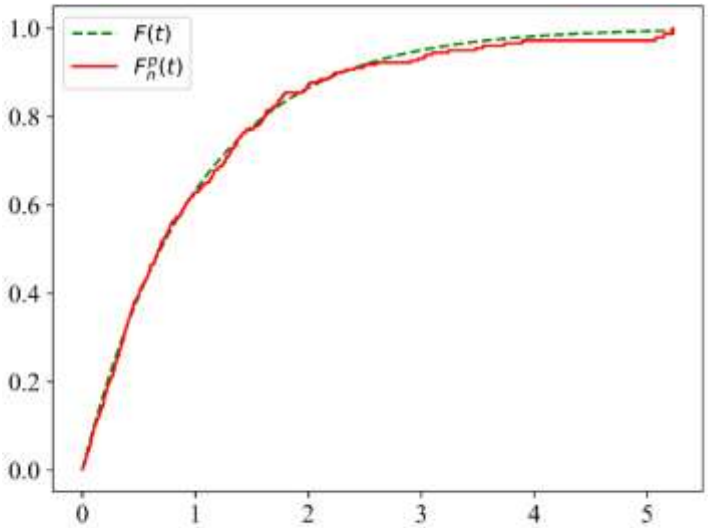
6-rasm. $F_n^{PR}(t)-$ Oldindan silliqlangan darajadagi tavakkalliklar nisbati bahosi

[13] da PRRP bahosining asimptotik xossalari ko‘rsatilgan. PRRP bahosi t dagi mustaqil va bir xil taqsimlangan tasodifiy funksiyalar yig‘indisi orqali $n \to \infty$ da nolga teng bo‘lgan qoldiq qiymatini ko‘rsatilgan. PRRP bahosining kuchli asosli barqarorligini ko‘rsatilgan. PRRP bahoning asimptotik normalligi ko‘rsatilgan.

[14] da PL - bahoning oldindan silliqlangan bahosi taklif etilgan bo‘lib, u (2) dagi $\delta_{(j)}$ ni (10) ning berilgan nuqtadagi qiymati bilan almashtirish natijasida olingan:

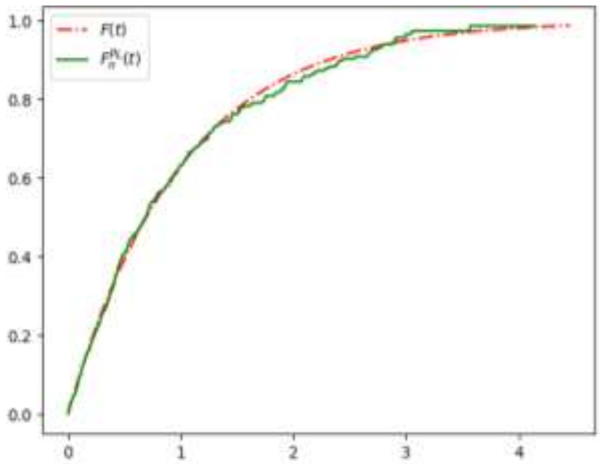
$$F_n^{p}(t) = 1 - \prod_{\{j:Z_{(j)} \le t\}} \left(1 - \frac{p_n\left(Z_{(j)}\right)}{n - j + 1} \right), \quad t \in Ў. \tag{13}$$

Endi yuqoridagi (13) bahoni qay darajada ekanligini o‘rganish maqsadida kompyuter modellashtirish yordamida ushbu model bo‘yicha tanlanma beradigan Pythonda dastur tuzib, bahoni chizishda bosh to‘plam taqsimoti sifatida $F(t) = 1 - e^{-t}$, $t \ge 0$, -eksponensial taqsimotni olib, undan $n = 500$ hajmli senzurlanish darajasi 30% (150 ta) bo‘lgan sun’iy tanlanma olamiz va u yordamida quyidagi chizmani chizib ko‘ramiz (7-rasm).

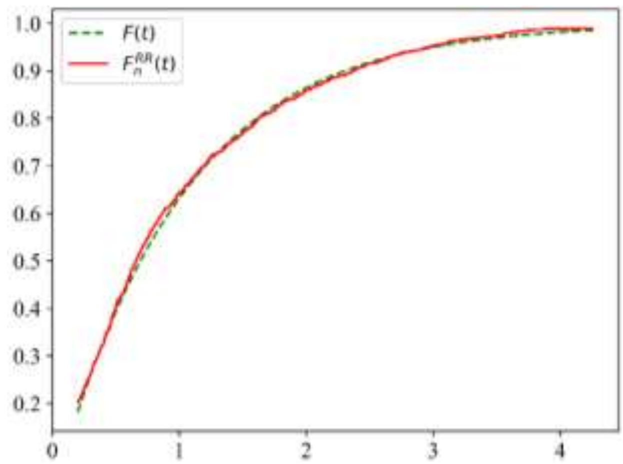


7-rasm. $F_n^p(t)$ – Kaplan-Mayerning silliqlangan bahosi

Endi biz noparametrik va yarim parametrik statistik baholarni sonli usullar yordamida tadqiq qilamiz. Quyidagi 8-rasm va 9-rasmlarda $n = 500$ hajmdagi modellashtirilgan tanlanma uchun parametri $\lambda = 1$ bo'lgandagi eksponensial taqsimoti uchun mos raravishda Kaplan-Mayerning $F_n^{PL}(t)$ – bahosi (2), (8-rasm**)** va Abdushukurovning $F_n^{RR}(t)$ – baholari (3) (9-rasm**)** chizmalari keltirilgan.



8-rasm. $F_n^{PL}(t)$ – bahosi



9-rasm. $F_n^{RR}(t)$ – bahosi

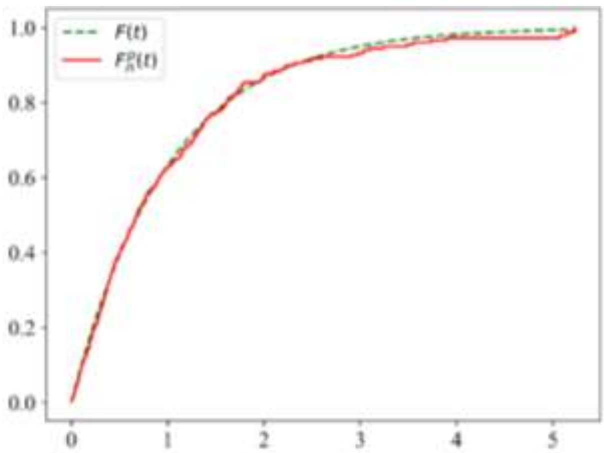
Agar har ikki baho uchun $\sum_{i=1}^{n-1}\left(F_n^{PL}(Z_i) - F(Z_i)\right)^2 = \Delta^{PL}$ hamda $\sum_{i=1}^{n-1}\left(F_n^{RR}(Z_i) - F(Z_i)\right)^2 = \Delta^{RR}$ o'rtacha kvadratik xatoliklarni belgilab olsak, u holda hisoblardan ko‘rsatib turibtiki, $F_n^{RR}(t)$ – baho $F_n^{PL}(t)$ – bahoga nisbatan samarador ekan.

$F_n^{RR}(t)$ – **bahoning** $F_n^{PL}(t)$ – **bahoga nisbatan samaradorligi jadvali**

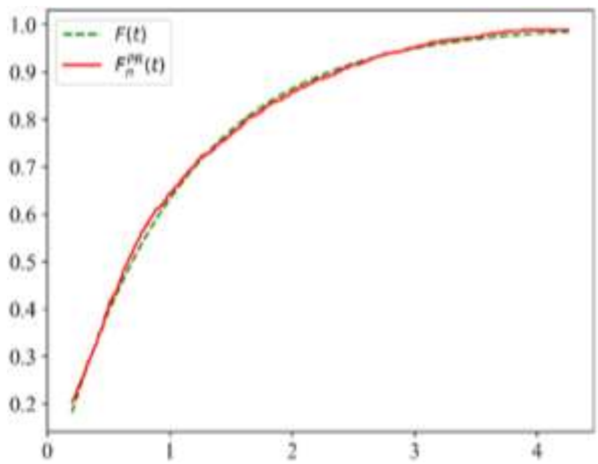
1-jadval

Yig‘indi	$\sum_{i=1}^{n-1}\left(F_n^{PL}(Z_i)-F(Z_i)\right)^2$	$\sum_{i=1}^{n-1}\left(F_n^{RR}(Z_i)-F(Z_i)\right)^2$	Samaradorligi
qiymati	0.204858	0.123901	0.604814

Xuddi shunday yuqoridagidek, (13) va (12) formulalar bilan beriladi Kaplan-Mayerning silliqlangan $F_n^{P}(t)$ bahosi va Abdushukurov-Nurmukhamedova-Bozorovlarning oldindan silliqlangan $F_n^{PR}(t)$ baholarining 10-rasm va 11-rasmlardagi chizmalarini ko‘rish mumkin.



10-rasm. $F_n^{P}(t)$ – bahosi



11-rasm. $F_n^{PR}(t)$ – bahosi

Ko‘rinib turibtiki, har ikki baho nazariy eksponensial taqsimotiga juda yaqin baholar ekan. Ammo, oyna kengligi $h(n)=n^{-\alpha}, 0<\alpha<1, n\in N$ ning turli qiymatlarida ham Oldindan silliqlangan darajadagi tavakkalliklar nisbati bahoning Kaplan-Mayerning silliqlangan bahoga nisbatan samarador ekan. Bu sonlar 2-jadvalda keltirilgan.

$F_n^{PR}(t)$ – **bahoning** $F_n^{P}(t)$ – **bahoga nisbatan samaradorligi jadvali**

2-jadval

Oyna kengligi	$\sum_{i=1}^{n-1}\left(F_n^{P}(Z_i)-F(Z_i)\right)^2$	$\sum_{i=1}^{n-1}\left(F_n^{PR}(Z_i)-F(Z_i)\right)^2$	Samaradorligi

$h(n)=(n)^{-1/2}$	0.294402	0.277757	0.943462
$h(n)=(n)^{-1/3}$	0.23152	0.210734	0.910219
$h(n)=(n)^{-1/5}$	0.181307	0.178650	0.985345

2-jadvaldagi hisoblardan koʻrinadiki, $F_n^{PR}(t)$ – Oldindan silliqlangan darajadagi tavakkalliklar nisbati bahosi, $F_n^{p}(t)$ – Kaplan-Mayerning silliqlangan bahosidan samarador ekan.

III. XULOSA

Tanlanma oʻng tomondan tasodifiy senzurlanganida senzor taqsimot ma'lum va noma'lum boʻlgan hollarda noma'lum taqsimot funksiyasini baholash koʻrilgan. Sezurlanuvchi $F(t)$ taqsimot uchun noparametrik koʻpaytma koʻrinishidagi baho $F_n^{PL}(t)$ – Kaplan va Mayer tomonidan, daraja koʻrinishidagi $F_n^{RR}(t)$ – tavakkalliklar nisbati bahosi Abdushukurov tomonidan, proporsional intensivlik modelidagi $F_n^{ACL}(t)$ – bahosini Abdushukurov-Cheng-Lin tomonidan va oldindan silliqlangan darajadagi tavakkalliklar nisbati $F_n^{PR}(t)$ – bahosi esa Abdushukurov Nurmukhamedova-Bozorovlar tomonidan taklif etilgan boʻlib, baholarni qay darajada ekanligini oʻrganish maqsadida kompyuter modellashtirish yordamida mos model boʻyicha tanlanma beradigan pythonda dastur tuzib, baholarni grafiklari chizib chiqilgan. **Noparametrik va yarim parametrik statistik baholarni sonli usullar yordamida taqqoslash** koʻrsatilgan. $F_n^{PL}(t)$ va $F_n^{RR}(t)$ har ikki baho nazariy eksponensial taqsimotiga juda yaqin baholar ekanligi va $F_n^{RR}(t)$ – bahoning $F_n^{PL}(t)$ – bahoga nisbatan samaradorligi aniqlangan. $F_n^{p}(t)$ va $F_n^{PR}(t)$ har ikki baho nazariy eksponensial taqsimotiga juda yaqin baholar ekanligi va oyna kengligi $h(n)$ ning turli qiymatlarida ham $F_n^{PR}(t)$ – oldindan silliqlangan darajadagi tavakkalliklar nisbati bahoning $F_n^{p}(t)$ – Kaplan-Mayerning silliqlangan bahosiga nisbatan samarador ekanligi koʻrsatilgan.

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