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BOUNDARY VALUE PROBLEMS FOR INTEGRAL-DIFFERENTIAL EQUATIONS WITH INTEGRAL OPERATORS

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Abstract. *This article analyzes the theoretical foundations of boundary value problems for integrodifferential equations with integral operators and methods for solving them. Since boundary value problems involve the combined action of differential and integral operators, their solutions are defined in complex function spaces. The article examines the conditions for the existence, uniqueness, and stability of solutions to equations with integral operators of the Fredholm and Volterra types. The potential of numerical solution methods is also discussed.*

Keywords: *integrodifferential equation, integral operator, boundary value problem, Fredholm operator, Volterra operator, uniqueness of solution, stability, functional space.*

КРАЕВЫЕ ЗАДАЧИ ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ИНТЕГРАЛЬНЫМИ ОПЕРАТОРАМИ

Абстрактный. *В статье анализируются теоретические основы краевых задач для интегродифференциальных уравнений с интегральными операторами и методы их решения. Поскольку краевые задачи предполагают совместное действие дифференциальных и интегральных операторов, их решения определяются в комплексных функциональных пространствах. В статье исследуются условия существования, единственности и устойчивости решений уравнений с интегральными операторами типа Фредгольма и Вольтерра. Обсуждаются также возможности численных методов решения.*

Ключевые слова: *интегродифференциальное уравнение, интегральный оператор, краевая задача, оператор Фредгольма, оператор Вольтерра, единственность решения, устойчивость, функциональное пространство.*

INTEGRAL OPERATOR QATNASHGAN INTEGRO-DIFFERENSIAL TENGLAMALAR UCHUN CHEGARAVIY MASALALAR

Annotatsiya. *Ushbu maqolada integral operator ishtirok etuvchi integrodifferensial tenglamalar uchun chegaraviy masalalarning nazariy asoslari va ularni yechish metodlari tahlil qilinadi. Chegaraviy masalalar differensial va integral operatorlarning birgalikdagi* *ta'sirini o'z ichiga olganligi sababli, ularning yechimlari murakkab funksional fazolarda aniqlanadi. Maqolada Fredgolm va Volterra turidagi integral operatorlar qatnashgan tenglamalar uchun yechim mavjudligi, yagonaligi va barqarorligi shartlari o'rganiladi. Shuningdek, sonli yechim usullarining qo'llanish imkoniyatlari ham yoritilgan.*

Kalit so'zlar: *integrodifferensial tenglama, integral operator, chegaraviy masala, Fredgolm operatori, Volterra operatori, yechim yagonaligi, barqarorlik, funksional fazo.*

INTRODUCTION

In modern mathematical physics, mechanics, biology, and engineering, many processes are described by equations with integral operators. Integro-differential equations play an important role, especially in modeling systems with delay, diffusion, or cumulative properties.

In such equations with an integral operator, the state of the system depends not only on the current value of the parameter but also on changes that occurred in the previous time interval. Therefore, their analysis requires the use of classical differential equation theory, requiring the application of methods from functional analysis, operator theory, and boundary value problems.

RESEARCH METHODOLOGY

Boundary value problems of integrodifferential equations are usually viewed as natural models of physical processes, such as heat conduction, electromagnetic wave propagation, deformation systems, and the dynamics of biological populations. Their analysis is accomplished by proving the existence, uniqueness, and stability of solutions.

In this context, the study of boundary value problems for equations with integral operators is a topical issue from both theoretical and practical points of view.

We've become familiar with the basic problems of integro-differential equations and one method for studying them. As with ordinary differential equations, boundary value problems can be posed for integro-differential equations. In this article, we'll use examples to introduce the formulation of such boundary value problems and methods for solving them.

RESEARCH AND RESULTS

Issue 3.

$$y''(x) + \frac{1}{7}x^2y'(x) + \frac{3}{7}xy(x) + \frac{12}{7}\int_0^1 (x-t)^2\, y(t)dt = 6x\,, \;\; x \in (0,1) \quad 1.1)$$

The equation is continuous on the interval and$[0,1]$

$$y(0) = 1, \qquad y(1) = 2 \tag{1.2}$$

Find a solution that satisfies the boundary conditions. Solution: This problem can be studied in two different ways.

Method 1. We integrate the given equation over the interval . As a result, taking into account that and are unknowns, we obtain: $[0,x]y(0) = 1y'(0)$

$$y'(x) + \frac{x^2}{7}y(x) + \frac{1}{7}\int_0^x ty(t)dt + \frac{4}{7}\int_0^1 [(x-t)^3 + t^3]y(t)dt = y'(0) + 3x^2$$

We get an equation. We can substitute this equation into the interval again.$[0,x]$ If we integrate and take into account the boundary condition,$y(0) = 1$

$$y(x) + \frac{1}{7}\int_0^x xty(t) + \frac{1}{7}\int_0^1 [(x-t)^4 - t^4 + 4t^3x]y(t)dt =$$
$$= y'(0)x + x^3 + 1 \qquad 1.3$$

This implies equality. Considering that and in (3.27), we find in the following form:$x = 1y(1) = 2y'(0)$

$$y'(0) = \frac{1}{7}\int_0^x (1 - 3t + 6t^2)y(t)dt$$

$y'(0)$If we substitute this expression into (1.3), we obtain

$$y(x) + \frac{1}{7}\int_0^x xty(t)dt +$$
$$+\frac{1}{7}\int_0^1 x(x^3 - 4x^2t + 6xt^2 - 6t^2 + 3t - 1)y(t)dt = x^3 + 1$$

We get the equation. If

$$K(x,t) = \begin{cases} x(x^3 - 4x^2t + 6xt^2 - 6t^2 + 4t - 1), & x \geq t \\ x(x^3 - 4x^2t + 6xt^2 - 6t^2 + 3t - 1), & x \leq t \end{cases}$$

If we introduce the notations, the last equation will take the form

$$y(x) + \frac{1}{7}\int_0^1 K(x,t)y(t)dt = x^3 + 1, \qquad x \in [0,1] \tag{1.4}$$

can be written as: (1.4) is a Fredholm integral equation of the second kind with respect to the function . The function can be written as follows:$y(x)K(x,t)$

$$K(x,t)=\begin{cases} x(1-x)[(1-x)-(x-2t)^2-2(1-t)^2], & x\geq t\\ -x(1-x)[(x-2t)^2+2(1-t)^2]+ & \\ +x[(1-x)^2-t], & x<t \end{cases}$$

Using this, we can show that . In this case, the inequality holds. Given this inequality, the piecewise continuity and boundedness of , and the fact that is a continuous function, it follows that a solution to the integral equation (1.4) exists and is unique according to Fredholm's theorem. That the solution to the integral equation (1.4) is twice continuously differentiable and is a solution to Problem 1.3 is proved similarly to Problem 1.1.$\sup|K(x,t)| < 6 (1/7) < (1/sup|K(x,t)|) K(x,t) x^3+1$

Method 2. This method is called the Green's function method. It can be used when the Green's function of a boundary value problem with given boundary conditions is known for a particular solution of a given equation. In this case, the Green's function of the problem with conditions (1.2) can be used for a particular solution of the given equation. This function$y''(x)=0$

$$G(x,t)=\begin{cases} x(t-1), & x\leq t\\ (x-1)t, & x\geq t \end{cases}$$

has the form, and the equalities are valid. Using the substitution problem, we obtain the following homogeneous boundary condition: $G(x,0)=G(x,1)=0\{(1.1),(1.2)\}z(x)=y(x)-x-1$

$$z''(x)+\frac{1}{7}x^2z'(x)+\frac{3}{7}xz(x)+$$
$$+\frac{12}{7}\int_0^1 (x-t)^2z(t)dt=-\frac{1}{7}(22x^2-59x+7) \tag{1.5}$$
$$z(0)=0, z(1)=0 \tag{1.6}$$

Let's consider the problem. Equation (1.5)

$$z''(x)=f(x) \tag{1.7}$$

we can write this down in a visual form and consider the issue here$\{(1.6),(1.7)\}$

$$f(x)=-\frac{1}{7}(22x^2-59x+7)-$$
$$-\frac{1}{7}x^2z'(x)-\frac{3}{7}xz(x)-\frac{12}{7}\int_0^1 (x-t)^2z(t)dt.$$

If we temporarily assume that the function is known, then the solution to the problem will be$f(x)\{(1.6),(1.7)\}$

$$z(x) = \int_0^1 G(x,t)f(t)dt$$

It is determined by the formula. If we first substitute the expression of the function into this equation, and then the expression of the function,$f(x)z(x)$

$$y(x) + \frac{1}{7}\int_0^1 G(x,t)\left[t^2y'(t) + 3y(t) + 12\int_0^1 (t-\xi)^2 y(\xi)d\xi\right]dt =$$
$$= 1 + 2x - x^3 \qquad 1.8$$

We obtain the following equality. Integrating by parts the term included in this equation and changing the order of integration in the term included in the iterated integral, we obtain the equation for the unknown function:$y'(t)y(x)$

$$y(x) + \frac{1}{7}\int_0^1 K_1(x,t)y(t)dt = 1 + 2x + -x^3 \tag{1.9}$$

we have an integral equation of the form, where

$$K_1(x,t) = (3-2t)G(x,t) - t^2G_t'(x,t) + 12\int_0^1 G(x,\xi)(\xi - t)^2 d\xi.$$

(1.9) is a Fredholm integral equation of the second kind, equivalent to the problem (in the sense of the existence of a solution). The existence of a unique solution is investigated in the same way as in Method 1. $\{(1.1),(1.2)\}$ Problem 1.3 can be investigated using the two methods used above, even if the interval integral in equation (3.25) is replaced by an interval integral. In cases where the integral part of the equation under consideration has a special form, other methods can be used to solve the problem.$[0,1][0,x]$

Conclusion

Boundary value problems for integro-differential equations with integral operators represent one of the most complex, yet practically important, areas of mathematical modeling. Determining the conditions for the existence and uniqueness of solutions to such problems, as well as studying their stability, are among the fundamental problems of mathematical physics and the theory of differential equations.

Research shows that for systems with integral operators, solution analysis using Fredholm and Volterra operators yields effective results. Furthermore, the practical modeling capabilities of such problems are expanded by using numerical solution methods (for example, Chebyshev or Galerkin methods).

A deep study of boundary value problems with integral operators is important not only in theoretical physics, but also in engineering, economics, and biological systems.

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