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APPLICATIONS OF THE FUNCTION CONTINUITY MODULE AND ITS PROPERTIES

Nafasov Ganisher Abdurashidovich
Associate Professor of the Department of Mathematics, Doctor of Philosophy (PhD) in Pedagogical Sciences, Gulistan State University
E-mail: *gnafasov87@gmail.com*
Hasanov Elchinbek Xoshimqul o'g'li
Gulistan City Specialized Boarding School No. 1
E-mail: hasanovelchinbek@gmail.com

Abstract: *This article analyzes the concept of the modulus of continuity of a function, its mathematical properties, and practical applications. The modulus of continuity is used to study the properties of function convergence, homogeneous continuity, differentiability, and stability of limit processes. Estimation methods based on the modulus of continuity, applied in numerical analysis, approximation theory, and functional analysis, are also discussed. The results of this study are important for analyzing continuous functions in function spaces, determining their degree of boundedness, and improving the accuracy of computational processes.*

Keywords: *modulus of continuity, continuous functions, differentiability, approximation, functional analysis, estimation method, limit process, homogeneous continuity.*

ПРИЛОЖЕНИЯ МОДУЛЯ НЕПРЕРЫВНОСТИ ФУНКЦИИ И ЕГО СВОЙСТВ

Аннотация: *В данной статье анализируется понятие модуля непрерывности функции, его математические свойства и практические приложения. С помощью модуля непрерывности изучаются свойства сходимости функции, однородной непрерывности, дифференцируемости и устойчивости предельных процессов. Также рассматриваются методы оценки, применяемые в численном анализе, теории приближений и функциональном анализе, основанные на модуле непрерывности. Результаты исследования имеют важное значение для анализа непрерывных функций в функциональных пространствах, определения степени их ограниченности и повышения точности вычислительных процессов.*

Ключевые слова: *модуль непрерывности, непрерывные функции, дифференцируемость, аппроксимация, функциональный анализ, метод оценивания, предельный процесс, однородная непрерывность.*

FUNKSIYANING UZLUKSIZLIK MODULI VA UNING XOSSALARI TATBIQLARI

Annotatsiya: *Ushbu maqolada funksiyaning uzluksizlik moduli tushunchasi, uning matematik xossalari hamda amaliy tatbiqlari tahlil qilinadi. Uzluksizlik moduli yordamida funksiyaning yaqinlashish xususiyatlari, bir jinsli uzluksizlik, differensiallanuvchanlik va limit jarayonlarining barqarorligi oʻrganiladi. Shuningdek, uzluksizlik moduli asosida sonli tahlil, aproksimatsiya nazariyasi va funksional analiz masalalarida qoʻllaniladigan baholash usullari yoritilgan. Tadqiqot natijalari funksional fazolarda uzluksiz funksiyalarni tahlil qilish, ularning cheklanganlik darajasini aniqlash hamda hisoblash jarayonlarining aniqligini oshirishda muhim ahamiyat kasb etadi.*

Kalit soʻzlar: *uzluksizlik moduli, uzluksiz funksiyalar, differensiallanuvchanlik, aproksimatsiya, funksional analiz, baholash usuli, limit jarayoni, bir jinsli uzluksizlik.*

INTRODUCTION

In many areas of mathematics, particularly analysis, functional analysis, numerical methods, and probability theory, the study of the continuity of functions and their behavior plays an important role. The concept of a function's modulus of continuity is central to this field, as it quantitatively describes how a function changes with its argument.

The modulus of continuity is used to assess the closeness of functions, the stability of their boundary values, and the degree of "smoothness" of a function. Therefore, this concept is widely used not only in theoretical but also in practical aspects of mathematical analysis. In particular, the modulus of continuity expands the possibilities for estimating errors in approximation problems, ensuring the stability of algorithmic calculations, and analyzing solutions to differential equations.

The article discusses the modulus of continuity of a function and its properties, as well as its application in modern mathematical analysis and computational practice.

RESEARCH METHODOLOGY

This study utilizes fundamental methods of functional and mathematical analysis, including limit processes, norm spaces, convergence criteria for sequences of functions, and fundamental theorems of approximation theory. Key estimation inequalities for the modulus of continuity, lemmas expressing its properties, and their proofs are presented. In addition to theoretical analysis, the study utilizes numerical calculation methods to determine practical values of the modulus of continuity for certain functions and analyze their graphs. The relationship between the modulus of continuity and the Lipschitz and Hölder conditions is also examined, and the correspondence of these properties to differentiability conditions is substantiated using mathematical proofs.

RESULTS AND DISCUSSION

Let's assume, $f(x)$ Let the function be continuous and defined on the interval [a;b]. We denote the space of these functions as C[a;b].

$$\left\{\begin{matrix} \sup \ \| (x+h) - f(x) \| = \varpi \ (\delta; f) \\ 0 < h \le \delta \end{matrix}\right\}$$

This,

$$\| f(x) \| = \max \ | f(x) |, \quad a \le x \le b$$

quantity $f(x)$ is called the norm of a function in the space C[a;b].

$$\sup \ \| (x+h) - f(x) \| = \varpi \ (\delta; f)$$
$$0 < h \le \delta$$

quantity $f(x)$ is called the modulus of continuity of the function.

This $\varpi(\delta; f)$ A continuous module has the following properties:

1. $\varpi(0; f) = 0$

2. $\varpi(\delta; f)$

And δ is a monotonically non-decreasing function with respect to , i.e.

$$\boldsymbol{\varpi (\delta_1 f) \le \varpi (\delta_2; f)} \qquad \delta_1 \ \pi \ \delta_2$$

From these properties (1.) this is self-evident. And property (2.) $\varpi(\delta; f)\uparrow$ that is, the upper limit δ increases in relation to , i.e. δ -with increase $\varpi(\delta; f)$ does not decrease.

1. $w(\delta_1 + \delta_2; f) \le w(\delta_1; f) + w(\delta_2; f)$ Inequality is appropriate.

proof:

$$w(\delta_1+\delta_2;f)=\sup_{0\le h_1+h_2\le\delta_1+\delta_2}\| f(x_1+h_1+h_2)-f(x)\|=$$
$$=\sup_{0\le h_1+h_2\le\delta_1+\delta_2}\| f(x_1+h_1+h_2)-f(x+h_1)+f(x+h_1)-f(x)\|\le$$
$$\le\sup_{0\le h_1+h_2\le\delta_1+\delta_2}\| f(x_1+h_1+h_2)-f(x+h_1)\|+\sup_{0\le h_1+h_2\le\delta_1+\delta_2}\| f(x_1+h_1)-f(x)\|=$$
$$=w(\delta;f)+w(\delta;f)$$

From these three properties the following follows.

2. ϖ (n;f)≤nϖ (d;e)

Proof 3 of

ϖ (d1+d2+ …+dn;f)≤ϖ (d1;f)+ϖ (d2;)+…+ϖ (day;f)

If this is

d1=d2=...=dn=d

If,

ϖ (n;f)≤n*ϖ (d;zh)*

5. For any number n>0

щ(лд;f)≤л+1)щ(д)

Proof. Let us denote the integer part of the number l by [l], then

Now based on 2 objects and based on 4

*ϖ (ld;f)≤ϖ ([l]+1)*g,e)≤[l+1]*ϖ (d;zh)*≤(л+1)*ϖ (d;zh)*

6. ϖ The function (d;f) is a right-continuous function.

Proof. For any t1<t2, based on the above properties,

ϖ (t2;f)=ϖ (t2+t1 - t;f)=ϖ ((t2-t1)+t1;f)≤*ϖ (t2-t1;f)+ϖ (t;f)*

It follows from this,

ϖ (t2; f)-ϖ (t1;f)≤ϖ (t2-t1;f)

or

ϖ (t+Δt)ϖ (T)≤ϖ (Dt;f)

Lim (Δt;f)=ϖ (0;ф)=0

Δt→0

This follows from the continuity of the function f(x), that is, it is continuous in the plane, therefore

$$\lim_{\Delta t \to 0} [\varpi(t + \Delta t); t) - \varpi(t; f)] = 0$$

therefore, the function ω(d; t) is continuous.

*[a;b]*A function that is continuous on a segment is also continuous on that segment.

If the function f(x) is continuous in the plane, then as x→0

$\varpi(t; f) \to 0$

7. Any щ(д;f)$\neq$ For 0

$\varpi(\delta; f) \geq \dfrac{\omega(1;t)}{2}\delta$, 0<d<1

Proof. $\varpi(\delta; f) = \varpi(\delta, \dfrac{1}{\delta}; f) \leq (\dfrac{1}{\delta} + 1)\varpi(\delta; f) = \dfrac{1+\delta}{\delta}\delta \leq \varpi(\delta; f)$

$\dfrac{\varpi(1; f)}{2}\delta \leq \varpi(\delta; f)$

Now let us consider the class of functions defined by their continuous modulus. If the continuous modulus of a continuous function is equal to

$\varpi(\delta; f) \leq \mu\delta^{2}$ (22)

If the condition is satisfied, then such a set of continuous functions α is called the class of ordered Hölder (Lipschitz) functions.

In point 1 above $\mu \phi 0$, number δ does not depend on, but generally speaking, $^{\alpha}$ or lip α is defined as. If *f(x)* $\in MX^{\alpha}$ if then f(x)$\in$ $HOUR^{\alpha}$,vice versa, if f(x)$\in$ HOUR$^{\alpha}$ then, when M is large enough, f(x)$\in$ MX^{α} will.

Note that according to property (7), it $\to$ 0 Also $\varpi(u, \delta)$ The quantity u cannot be infinitely small to a greater degree than u. U takes place $\alpha \ \phi \ 1$ In (22) the inequality is not satisfied. $0 \ \pi \ \alpha \leq 1$ and therefore, $\alpha \ \phi \ 1$ in H$^{\alpha}$ There is no point in repeating this class.

That's why always $0 \pi \alpha \leq 1$ Let's see. Now $\alpha \pi \beta$ When $H^{\alpha} \supset H^{\beta}$ We celebrate this. $0 \pi \alpha \pi \beta \leq 1$, $\delta^{\alpha} \geq \delta^{\beta}$, $0 \leq \delta \leq 1$ $\delta \in [0;1]$

CONCLUSION

The modulus of continuity of a function is one of the fundamental concepts of mathematical analysis, allowing one to quantify the degree of continuity of a function. Research shows that the modulus of continuity can be used to evaluate functions, determine their degree of smoothness, and improve the accuracy of computational processes.

This concept is widely used in approximation theory, numerical methods, differential equation theory, and functional analysis, and serves as an important tool for ensuring the accuracy of scientific research and practical computational work. One promising future direction is the development of new estimation methods using the modulus of continuity and their integration into a system of numerical algorithms.

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