DEVELOPMENT OF STUDENTS' LOGICAL THINKING USING STEREOGRAPHIC PROJECTION AND ITS APPLICATION IN GEOMETRY LESSONS

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Abstract: *This article analyzes the didactic potential of stereographic projection in teaching geometry and its role in developing students' logical thinking. The study provides a scientific perspective on the impact of the process of representing spatial figures on a plane using stereographic projection on students' analytical, logical, and creative thinking. It examines the mechanisms for using stereographic projection in the educational process and ways to improve the effectiveness of geometry lessons by combining it with modeling and interactive methods. The study's findings contribute to the expansion of students' spatial imagination using stereographic projection, as well as the development of their ability to logically analyze and independently find solutions to problematic situations.*

Keywords:*stereographic projection, geometric education, spatial imagination, logical thinking, educational activities, teaching methods, visual modeling, didactic capabilities.*

GEOMETRIYA DARSLARIDA STEREOGRAFIK PROYEKSIYA VA UNING TATBIQLARI QO'LLAB O'QUVCHILARNING MANTIQIY TAFAKKURINI RIVOJLANTIRISH

Annotatsiya: *Ushbu maqolada geometriya fanini o'qitishda stereografik proyeksiya metodining didaktik imkoniyatlari va uning o'quvchilarning mantiqiy tafakkurini rivojlantirishdagi o'rni tahlil qilinadi. Tadqiqotda stereografik proyeksiya asosida fazoviy shakllarni tekislikda tasvirlash jarayonining o'quvchilarda tahliliy, mantiqiy va ijodiy fikrlash ko'nikmalariga ta'siri ilmiy asosda yoritilgan. Shuningdek, stereografik proyeksiyaning o'qitish jarayonida qo'llanish mexanizmlari, uni modellashtirish va interfaol metodlar bilan uyg'unlashtirish orqali geometriya darslarining samaradorligini oshirish yo'llari ko'rib chiqiladi. Tadqiqot natijalari stereografik proyeksiya yordamida o'quvchilarning fazoviy tasavvurini kengaytirish, mantiqiy tahlil va muammoli vaziyatlarda mustaqil yechim topish qobiliyatini shakllantirishga xizmat qiladi.*

Kalit so'zlar: *stereografik proyeksiya, geometriya ta'limi, fazoviy tasavvur, mantiqiy tafakkur, o'quv faoliyati, o'qitish metodikasi, vizual modellashtirish, didaktik imkoniyatlar.*

РАЗВИТИЕ ЛОГИЧЕСКОГО МЫШЛЕНИЯ УЧАЩИХСЯ С ИСПОЛЬЗОВАНИЕМ СТЕРЕОГРАФИЧЕСКОЙ ПРОЕКЦИИ И ЕГО ПРИМЕНЕНИЯ НА УРОКАХ ГЕОМЕТРИИ

Абстрактный: *В статье анализируются дидактические возможности метода стереографической проекции в обучении геометрии и его роль в развитии логического мышления учащихся. В исследовании с научной точки зрения освещается влияние процесса изображения пространственных фигур на плоскости с помощью стереографической проекции на аналитическое, логическое и творческое мышление учащихся. Рассматриваются механизмы использования стереографической проекции в учебном процессе, пути повышения эффективности уроков геометрии путем ее сочетания с моделированием и интерактивными методами. Результаты исследования способствуют расширению пространственного воображения учащихся с помощью стереографической проекции, формированию умения логически анализировать и самостоятельно находить решения проблемных ситуаций.*

Ключевые слова: *стереографическая проекция, геометрическое образование, пространственное воображение, логическое мышление, учебная деятельность, методика обучения, наглядное моделирование, дидактические возможности.*

INTRODUCTION

In modern education, one of the main goals of geometry is to develop students' logical thinking and spatial imagination. Since geometry is the science that studies spatial figures, their relationships, and the laws of change, it plays an important role in developing students' analytical and logical thinking.

From this perspective, stereographic projection is a mathematical model expressing the relationship between spherical and flat geometry, based on the projection of points on a sphere onto a plane. It has great didactic value not only in the study of theoretical questions of geometry but also in practical exercises, promoting the development of spatial imagination, the ability to identify logical connections, and the skills of mutual analysis.

Today, by introducing stereographic projection into the learning process and combining it with interactive methods, students can:

Perception of the relative position of geometric objects;

logical analysis of connections between projections;

develop analytical thinking and reasoning skills;

You can develop the skills of independently drawing conclusions based on creative thinking.

Thus, the use of stereographic projection increases the activity of geometry lessons and allows students to connect theoretical knowledge with practical modeling.

RESEARCH METHODOLOGY

Definition 1.1. The geometric locus of points that are at the same distance from a given point in space is called a sphere.

Theorem 1.1. The intersection of a sphere with any plane is a circle.

Proof: Let us assume that the sphere and its intersection with the plane ABD Let a curved region be given (Fig. 1.1).

In this $OA = OB$ And ABD industry OC perpendicular. Also, $\angle OCA = \angle OCB$ Also, ΔOCA And ΔOCB for triangles OC From this we can conclude that ΔOCA And ΔOCB When triangles are similar $CA = CB$ We'll find out. A And B points ABD Based on the agricultural tariff for discretionary points of the sector ABD The field will have the shape of a circle. The theorem is proven.

Result 1.A straight line drawn from the center of a sphere to the center of a circle is perpendicular to the plane of the circle.

Result 2. Planes that pass at the same distance from the center of a sphere create equal areas. A plane that passes at a shorter distance from the center of a sphere creates a larger area.

Definition 1.4. The length of the largest circle connecting two points on a sphere is called the spherical distance between these points.

Theorem 1.2. The spherical distances from the pole of a circle on a sphere to all points of this circle are equal.

Proof. P_1P_2 points ABC polar points of the hall, A, B, C Let the dots be random notes of the circle. O_1 dot ABC Center of the courtyard. PA , PB , PC Let us prove their equality.

As you know, $O_1A = O_1B = O_1C$ And O_1P height ΔAO_1P , ΔBO_1P And ΔCO_1P is common to triangles. In this case, based on the equality of triangles, PA , PB , PC The lines are also equal to each other. These lines are considered to be equal chords of the sphere.

Therefore, $\angle AOP$, $\angle BOP$ And $\angle COP$ The angles are also equal, and these angles are perpendicular. PA , PB , PC It turns out they are also equal. The theorem is proven.

When creating images of figures in space, their projections onto a plane are used. To construct an image of a figure in a projection, one must select a point in space that will serve as the projection center, draw lines connecting this point to points of the projected figure, and find the intersection points of these lines with the given plane. The resulting image is called the projection of the figure onto the given plane.

If the projected figure is circular, its projection is the line of intersection of the surface formed by the lines passing through the projection center and the points on the circle with the given plane. Such a surface is called a cone of revolution. The section of this surface by the plane generally need not be circular: it can be a curve, an ellipse, a parabola, or a hyperbola, as long as the cutting plane does not pass through the apex of the cone.

However, there is a remarkable projection in which the image of a circle is always a circle or a line. If we consider only circles lying on a sphere and take one of the points on this sphere as the projection center, and the projection plane is the plane passing through the diametrically opposite point of the sphere, or an arbitrary plane parallel to it and not passing through the center of the sphere, we can obtain the aforementioned projection. This projection is called a stereographic projection. This projection is widely used in astronomy, various branches of mathematics, and geography.

RESEARCH AND RESULTS

Stereographic projection is a special case of inversion, which is an important tool for solving some complex problems in geometry..

Stereographic projection is a projection from a point S on a sphere to a point S' diametrically opposite this point. σ is called a flat projection.

If σ The properties of this projection will not change if the plane is replaced by a plane parallel to it. σ projection onto a parallel plane

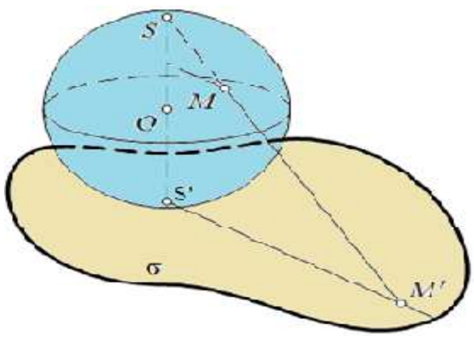

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Figure 1

It should not pass through the center. This plane is usually taken to be the diametrical plane of the sphere.

We will show the following three properties of stereographic projection:

A) A circle lying on a sphere σ It is depicted as a circle on a plane or as a straight line passing through the center of the projection of the circle (Fig. 2).

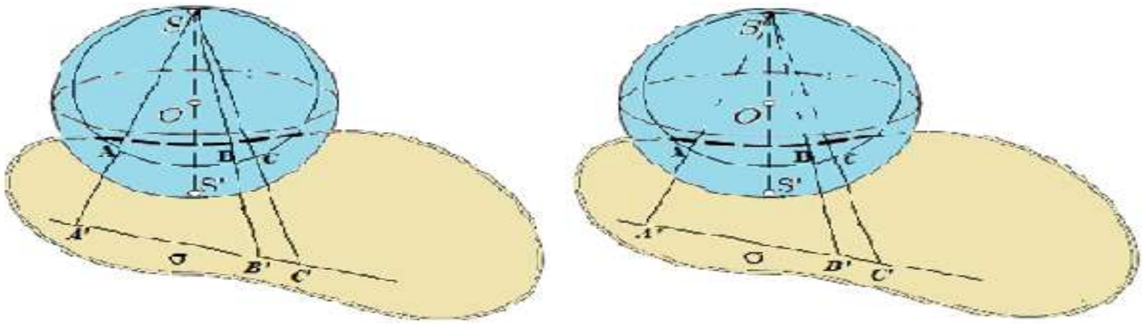

Figure 2

B) In stereographic projection, the angles between curves lying on the sphere are equal to the angles between the projections of these curves onto the plane.

C) If we describe an angle around a diameter passing through the pole of a sphere, σ In a plane, this angular rotation occurs around the point where the plane touches the sphere.

This property is not something that is in the sphere of direct use. M it's his matter M' go to projection SS' dia

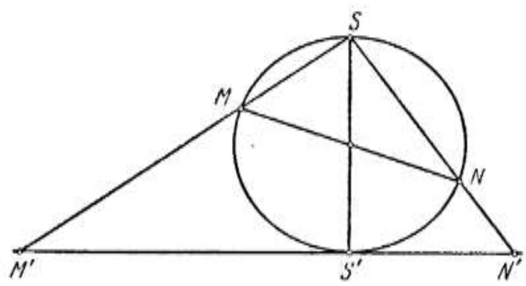

Figure 3

executed on a plane passing through a meter and a sphere φ on the corner SS' The line intersecting the projection plane with the plane passing through it also rotates by this angle.

You are truly a circle. MN stretched in diameter MSN Because the angle is right. $M^{'}SN^{'}$ in a triangle S The angle is also right. $SS^{'}$ The cross section is rectangular. $M^{'}SN^{'}$ Since the height of the triangle is equal to $S^{'}M^{'} \cdot S^{'}N^{'} = \left(SS^{'}\right)^{2}$ Equality is appropriate. In this equality $SS^{'}$ What $2R$ If we define it through (1), we get equality.

Now at point M' σ If there is an arbitrary point (different from S') on the plane, then there is a point M' $N^{'}$ We put the dot in the correct position: $M^{'}$ dot S We are in touch with, $M^{'}S$ intersection of a straight line with a sphere M we find the point, M diametrically opposite to the point $N^{'}$ We find it. Thanks to this, we know that the plane S differs from optional $M^{'}$ The point is located somewhere on this plane. $N^{'}$ We managed to make the exchange on the spot.

Stereographic projections are also used to depict the Earth's surface on a plane, that is, to create maps. On maps created using stereographic projections, the angles between curved lines, according to property (B), are equal to their natural values.

You can also derive the stereographic projection expression in coordinates. To do this: σ rectangular Cartesian coordinate system as a projection plane Oxy plane and center of projection S dot Oz Let it be located on the axis. In this case we see R equation of a sphere with radius

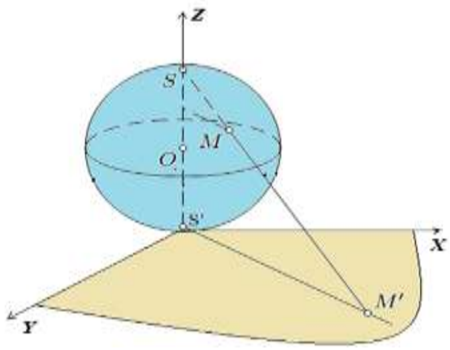

Figure 4

$x^{2} + y^{2} + \left(z - R\right)^{2} = R^{2}$ Stereographic projection is an arbitrary projection of a sphere. $M\left(x, y, z\right)$ dot Oxy airplane $M'\left(x', y', 0\right)$ Let us derive equations that connect the coordinates of these points, i.e. $M'(x', y', 0)$ your point of view x', y' coordinates $M(x, y, z)$ coordinates of the point x, y, z We find the expression through.

Obviously $\overrightarrow{SM}$ And $\overrightarrow{SM'}$ The vectors are collinear to each other. Therefore, $\overrightarrow{SM'} = k\overrightarrow{SM}$ equality and S coordinates of the point $S(0, 0, 2R)$ We will create the following taking into account the fact that

$$\begin{cases} x' = kx, \\ y' = ky, \\ 2R = k(2R - z). \end{cases}$$

(2) according to the third equality

$$k = \frac{2R}{2R - z}$$

from this $\begin{cases} x' = \dfrac{2Rx}{2R - z}, \\ y' = \dfrac{2Ry}{2R - z}, \\ z' = 0. \end{cases}$ (3) equality follows.

It is also possible to obtain the inverse substitution (3). For this, from equality (2) we have: $\begin{cases} x' = kx, \\ y' = ky, \\ 2R = k(2R - z). \end{cases} \Leftrightarrow \begin{cases} x = \dfrac{x'}{k}, \\ y = \dfrac{y'}{k}, \\ z = 2R(1 - \dfrac{1}{k}). \end{cases}$ (4) We will have equality. $M(x, y, z)$ Since the point belongs to the sphere, and using equation (4), we obtain the following $\left(\frac{x'}{k}\right)^2 + \left(\frac{y'}{k}\right)^2 + \left(2R(1 - \frac{1}{k}) - R\right)^2 = R^2$ We form equation (5). From equation (5) $k = \frac{1}{4R^2}\left(x'^2 + y'^2 + 4R^2\right)$ (6) we compose the equation. From equalities (6) and (4)

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$$\begin{cases} x = \dfrac{4R^2 x'}{x'^2 + y'^2 + 4R^2}, \\ y = \dfrac{4R^2 y'}{x'^2 + y'^2 + 4R^2}, \\ z = \dfrac{2R\left(x'^2 + y'^2\right)}{x'^2 + y'^2 + 4R^2}. \end{cases}$$

We form equality (7).

Conclusion

Scientific analysis shows that stereographic projection is not only an important concept in geometry but also an effective pedagogical tool for developing students' logical thinking. Using this method in the educational process leads to the following results:

the level of spatial imagination and analytical thinking of students increases;

students' independence in solving problems increases;

The ability to analyze the relationships between complex geometric objects using stereographic projection is developed;

In combination with digital tools, a culture of interdisciplinary thinking among students is developed.

Therefore, improving the methodology for teaching stereographic projection in geometry lessons and combining it with modern pedagogical technologies is of great importance in developing students' logical thinking, forming a culture of spatial thinking, and increasing their interest in the subject of geometry.

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