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KOMPLEKS ANALIZ METODLARI YORDAMIDA YIG'INDI VA KO'PAYTMALARNI HISOBLASH

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Annotatsiya: *Ma'lumki, matematika kursidagi ba'zi muhim yig'indilarni oddiy usullarda hisoblash birmuncha qiyinchiliklar tug'dirishi mumkin. Ammo, kompleks analiz usullaridan foydalangan holda bu yig'indilarni oson hisoblash mumkin. Ushbu maqolada ayrim yig'indilar va ko'paytmalarni kompleks analiz usullaridan foydalangan holda hisoblash amaliy jihatdan oson hal qilish mumkinligi ko'rsatilgan.*

Kalit so'zlar: *olimpiada masalalari, olimpiada, mantiqiy fikrlash, analitik geometriya, geometrik metod.*

ВЫЧИСЛЕНИЕ СУММ И УМНОЖЕНИЙ С ИСПОЛЬЗОВАНИЕМ МЕТОДОВ КОМПЛЕКСНОГО АНАЛИЗА

Абстрактный: *Известно, что вычисление некоторых важных сумм в курсе математики с использованием простых методов может вызывать определенные трудности. Однако, используя методы комплексного анализа, эти суммы можно легко вычислить. В данной статье показано, что вычисление некоторых сумм и множителей с использованием методов комплексного анализа может быть легко решено на практике.*

Ключевые слова: олимпиадные задачи, олимпиада, логическое мышление, аналитическая геометрия, геометрический метод.

CALCULATING SUM AND MULTIPLIES USING COMPLEX ANALYSIS METHODS

Annotation: *It is known that calculating some important sums in the mathematics course using simple methods can cause some difficulties. However, using complex analysis methods, these sums can be easily calculated. This article shows that calculating some sums and multiplies using complex analysis methods can be practically easily solved.*

Keywords: *Olympiad problems, Olympiad, logical thinking, analytical geometry, geometric method.*

KIRISH

Elementar algebra kursini o'rganish davomida sonlar sohasini kengaytira bordik. Butun musbat sonlar $\rightarrow$ butun musbat va manfiy sonlar $\rightarrow$ ratsional sonlar

$\rightarrow$ ratsional va irratsional sonlardan iborat haqiqiy sonlar. O'rta maktab kursidan ham ma'lumki har qanday haqiqiy koeffitsiyentli kvadrat tenglama ham haqiqiy sonlar sohasida yechimga ega bo'lavermaydi. Shu sababli ham haqiqiy sonlar sistemasini kengaytirishga zarurat tug'iladi.

Har qanday haqiqiy koeffisiyentli ko'phad haqiqiy sonlar sohasida ildizga ega bo'lavermasligi bu sonlar sistemasini kengaytirishga to'g'ri keladi. Kompleks sonlar to'plamining yopiqligi ya'ni, har qanday darajasi birdan kichik bo'lmagan ko'phad, albatta, bitta kompleks ildizga ega bo'lishi bu to'plamni o'rganishimizga asos bo'ladi.

Ma'lumki, matematika kursidagi ba'zi muhim yig'indilarni oddiy usullarda hisoblash birmuncha qiyinchiliklar tug'dirishi mumkin. Ammo, kompleks analiz usullaridan foydalangan holda bu yig'indilarni oson hisoblash mumkin.

TATQIQOTDA QO'LLANILGAN METODLAR

Ba'zi hisoblash murakkab bo'lgan yig'indi va ko'paytmalarning qiymatlarini kompleks analiz metodlaridan foydalangan holda hisoblash va oson hisoblashga qulaylik tug'diradigan yo'llarini topish masalasining o'rganilishi kompleks sonlar nazariyasining zamonaviy kontseptsiyalarini osonroq tushunishga yordam beradi va shuningdek, kompleks analizning tatbiqlari bobining rivojlanishiga asos bo'ladi hamda haqiqiy analizning ba'zi masalalarini hal qilish usullarini yanada ko'paytiradi.

Mazkur ishda kompleks sonlar xossalaridan foydalanib, Muavr va N'yuton formulalarini qo'llash natijasida isbotlash mumkin bo'lgan ba'zi bir ayniyatlar keltirilgan va kompleks sonlarning yig'indi ko'paytmalarni hisoblashdagi tatbiqlari ko'rsatilgan.

OLINGAN NATIJALAR VA ULARNING TAHLILI

1. Kompleks sonlar yordamida ayniyatlarni isbotlash.

1. Trigonometrik ko'rinishda berilgan kompleks sonni darajaga ko'tarishning ushbu

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi) \qquad (1)$$

formulasida $r = 1$ deb olsak, u holda Muavr formulasining xususiy holi, ya'ni

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi \qquad (*)$$

tenglikni hosil qilamiz. Bu formulaning chap tomonidagi qavsni N'yuton binomi formulasi bo'yicha ochib chiqib, so'ngra $i^2 = -1,\ i^3 = -i,\ \ i^4 = 1$ va umuman ixtiyoriy musbat butun k son uchun

$$i^{4k} = 1,\ \ i^{4k+1} = i,\ \ i^{4k+2} = -1,\ \ i^{4k+3} = -i \qquad (2)$$

ekanligini hisobga olsak va (*) tenglikning ikkala tomonidagi kompleks sonlarning haqiqiy va mavhum qismlarini tenglashtirib quyidagi formulalarni hosil qilamiz:

$$\cos n\varphi = \cos^n \varphi - C_n^2 \cos^{n-2}\varphi \sin^2\varphi + C_n^4 \cos^{n-4}\varphi \sin^4\varphi - \dots ,$$

$$\sin n\varphi = C_n^1 \cos^{n-1}\varphi \sin\varphi - C_n^3 \cos^{n-3}\varphi \sin^3\varphi + C_n^5 \cos^{n-5}\varphi \sin^5\varphi - \dots$$

bu yerda $C_n^k = \dfrac{n(n-1)(n-2)....(n-k+1)}{1\cdot 2\cdot 3\cdot \dots \cdot k}$.

Shunday qilib, biz karrali burchakning kosinus va sinuslarini oddiy burchak kosinus va sinuslari orqali ifodalovchi formulalarni hosil qildik. Xususiy hollarni qaraylik.

$n = 2$ bo'lganda bizga elementar matematikadan ma'lum bo'lgan quyidagi formulalarni hosil qilamiz:

$$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi,$$

$$\sin 2\varphi = 2\sin\varphi \cdot \cos\varphi .$$

$n = 3$ bo'lganda esa quyidagi formularni hosil qilamiz:

$$\cos 3\varphi = \cos^3\varphi - 3\cos\varphi \sin^2\varphi,$$

$$\sin 3\varphi = 3\cos^2\varphi \sin\varphi - \sin^3\varphi.$$

2. $\sin^3 x$ ni birinchi darajali kosinus va sinuslarning karralisi orqali ifodalang.

Yechish. $\alpha = \cos x + i\sin x$ deb belgilash kiritaylik. U holda ushu belgilashdan

$$\alpha^{-1} = \frac{1}{\alpha} = \frac{1}{\cos x + i\sin x} = \frac{\cos x - i\sin x}{(\cos x + i\sin x)(\cos x - i\sin x)} =$$

$$= \frac{\cos x - i\sin x}{\cos^2 x + \sin^2 x} = \cos x - i\sin x.$$

U holda

$$\alpha^{k}=(\cos x+i\sin x)^{k}=\cos kx+i\sin kx,$$

$$\alpha^{-k}=(\cos x+i\sin x)^{-k}=[(\cos x+i\sin x)^{-1}]^{k}=[\cos(-x)+i\sin(-x)]^{k}=$$

$$=\cos(-kx)+i\sin(-kx)=\cos kx-\sin kx.$$

Shunday qilib,

$$\alpha^{k}=\cos kx+i\sin kx,$$

$$\alpha^{-k}=\cos kx-i\sin kx.$$

Bu yerdan

$$\cos kx=\frac{\alpha^{k}+\alpha^{-k}}{2},\qquad \sin kx=\frac{\alpha^{k}-\alpha^{-k}}{2i}. \tag{3}$$

formulalarni hosil qilamiz. Ushbu formulada $k=1$ deb olsak, u holda

$$(\sin x)^{3}=\left(\frac{\alpha-\alpha^{-1}}{2i}\right)^{3}=-\frac{1}{8i}(\alpha^{3}-3\alpha+3\alpha^{-1}-\alpha^{-3})=-\frac{1}{8i}[(\alpha^{3}-\alpha^{-3})-3(\alpha-\alpha^{-1})]=$$

$$=-\frac{1}{8i}[2i\sin 3x-3\cdot 2i\sin x]=\frac{1}{4}(3\sin x-\sin 3x).$$

Shunday qilib,

$$\sin^{3}x=\frac{3\sin x-\sin 3x}{4}.$$

3. $\cos^{8}x$ ni birinchi darajali kosinus va sinuslarning karralisi orqali ifodalang.

Yechish. (3) tenglikda $k=1$ deb quyidagi tenglikni hosil qilamiz:

$$(\cos x)^{8}=\left(\frac{\alpha+\alpha^{-1}}{2}\right)^{8}=\frac{1}{256}(\alpha^{8}+8\alpha^{6}+28\alpha^{4}+56\alpha^{2}+70+$$

$$+56\alpha^{-2}+28\alpha^{-4}+8\alpha^{-6}+\alpha^{-8})=\frac{1}{256}[(\alpha^{8}+\alpha^{-8})+8(\alpha^{6}+\alpha^{-6})+$$

$$+28(\alpha^{4}+\alpha^{-4})+56(\alpha^{2}+\alpha^{-2})+70]=\frac{1}{256}(2\cos 8x+16\cos 6x+$$

$$+56\cos 4x+112\cos 2x+70)=\frac{\cos 8x+8\cos 6x+28\cos 4x+56\cos 2x+35}{128}.$$

Demak,

$$\cos^8 x = \frac{\cos 8x + 8\cos 6x + 28\cos 4x + 56\cos 2x + 35}{128}.$$

4. Ushbu

$$\delta = 1 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - C_n^{10} + \ldots$$

yig'indini hisoblang.

Yechish. $(1+i)^n$ ni N'yuton formulasidan foydalanib yoyib yozamiz:

$$(1+i)^n = 1 + C_n^1 \cdot i + C_n^2 \cdot i^2 + C_n^3 \cdot i^3 + \ldots + C_n^{n-1} \cdot i^{n-1} + i^n$$

Bizga ma'lum (*) tengliklardan foydalanib ushbu tenglikni quyidagicha o'zgartiramiz:

$$(1+i)^n = \left(1 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - \ldots\right) + i\left(C_n^1 - C_n^3 + C_n^5 - C_n^7 + \ldots\right).$$

Ikkinchi tomondan,

$$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

ekanligini hisobga olsak, u holda

$$(1+i)^n = 2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right)$$

tenglikka kelamiz. Demak,

$$\delta = 1 - C_n^2 + C_n^4 - C_n^6 + C_n^8 - C_n^{10} + \ldots = 2^{\frac{n}{2}}\cos\frac{n\pi}{4},$$

$$C_n^1 - C_n^3 + C_n^5 - C_n^7 + C_n^9 - \ldots = 2^{\frac{n}{2}}\sin\frac{n\pi}{4},$$

tengliklarni hosil qilamiz. Xususan, $n = 14$ deb olsak, u holda

$$1 - C_{14}^2 + C_{14}^4 - C_{14}^6 + C_{14}^8 - C_{14}^{10} + C_{14}^{12} - C_{14}^{14} = 2^7\cos\frac{14\pi}{4} = 0,$$

$$C_{14}^1 - C_{14}^3 + C_{14}^5 - C_{14}^7 + C_{14}^9 - C_{14}^{11} + C_{14}^{13} = 2^7\sin\frac{14\pi}{4} = 128.$$

Umuman olganda,

agar $n = 4m$ bo'lsa, u holda $\delta = (-1)^m 2^{2m}$, agar $n = 4m+1$ bo'lsa, u holda $\delta = (-1)^m 2^{2m}$,

agar $n = 4m+3$ bo'lsa, u holda $\delta = (-1)^{m+1} 2^{2m+1}$,

agar $n = 4m+2$ bo'lsa, u holda $\delta = 0$.

5. $tg\,6x$ ni tgx orqali ifodalang.

Yechish. Ma'lumki,

$$tg\,6x = \frac{\sin 6x}{\cos 6x}.$$

shu sababli avvalo $\sin 6x$ va $\cos 6x$ larni $\sin x$ va $\cos x$ lar orqali ifodasini topamiz:

$$\sin 6x = 6\cos^5 x \sin x - 20\cos^3 x \sin^3 x + 6\cos x \sin^5 x,$$

$$\cos 6x = \cos^6 x - 15\cos^4 x \sin^2 x + 15\cos^2 x \sin^4 x - \sin^6 x.$$

Bulardan,

$$tg\,6x = \frac{6\cos^5 x \sin x - 20\cos^3 x \sin^3 x + 6\cos x \sin^5 x}{\cos^6 x - 15\cos^4 x \sin^2 x + 15\cos^2 x \sin^4 x - \sin^6 x} =$$

$$= \frac{6tgx - 20\,tg^3 x + 6tg^5 x}{1 - 15\,tg^2 x + 15\,tg^4 x - tg^6 x}.$$

Ushbu oxirgi tenglikni hosil qilish uchun yuqoridagi kasrning surati va maxrajini $\cos^6 x$ ga bo'ldik.

6. Quyidagi tenglikni isbotlang.

$$2\cos mx = (2\cos x)^m - \frac{m}{1}(2\cos x)^{m-2} + \frac{m(m-3)}{1\cdot 2}(2\cos x)^{m-4} - \ldots +$$

$$+ (-1)^p \frac{m(m-p-1)(m-p-2)\ldots(m-2p+1)}{p!}(2\cos x)^{m-2p} + \ldots \quad (4)$$

Yechish.

$$C_{m-p}^{p} + C_{m-p-1}^{p-1} = \frac{(m-p)(m-p-1)(m-p-2)\ldots(m-2p+1)}{p!} +$$

$$+ \frac{(m-p-1)(m-p-2)\ldots(m-2p+1)}{(p-1)!} = \frac{m(m-p-1)(m-p-2)\ldots(m-2p+1)}{p!}$$

bo‘ladi.

$$2\cos mx = S_m, \quad 2\cos x = a$$

belgilash kiritamiz, u holda

$$S_m = a^m - ma^{m-2} + (C_{m-2}^2 + C_{m-3}^1)a^{m-4} - \ldots -$$

$$-(-1)^p(C_{m-p}^p + C_{m-p-1}^{p-1})a^{m-2p} + \ldots$$

tenglikni hosil qilamiz. Ko‘rsatish qiyin emaski

$$2\cos mx = 2\cos x \cdot 2\cos(m-1)x - 2\cos(m-2)x,$$

yoki yuqoridagi belgilashimizga ko‘ra

$$S_m = aS_{m-1} - S_{m-2}.$$

Ko‘rish qiyin emaski,

$m = 1$ da isbotlanishi lozim bo‘lgan tenglik o‘rinli:

$$2\cos x = 2\cos x.$$

$m = 2$ bo‘lsin, u holda

$$2\cos 2x = (2\cos x)^2 - \frac{2}{1}(2\cos x)^0 = 4\cos^2 x - 2$$

Bu esa bizga elementar matematikadan ma’lum bo‘lgan ayniyatdir:

$$1 + \cos 2x = 2\cos^2 x.$$

Faraz qilaylik,

$$S_{m-1} = a^{m-1} - (m-1)a^{m-3} + (C_{m-3}^2 + C_{m-4}^1)a^{m-5} - \ldots -$$

$$-(-1)^p(C_{m-p-1}^p + C_{m-p-2}^{p-1})a^{m-2p-1} + \ldots,$$

$$S_{m-2} = a^{m-2} - (m-2)a^{m-4} + (C_{m-4}^2 + C_{m-5}^1)a^{m-6} - \ldots -$$

$$-(-1)^{p-1}(C_{m-p-1}^{p-1} + C_{m-p-2}^{p-1})a^{m-2p} + \ldots.$$

U holda

$$S_m = aS_{m-1} - S_{m-2} = a^m - ma^{m-2} + \ldots +$$

$$+(-1)^p(C_{m-p-1}^p + C_{m-p-2}^{p-1} + C_{m-p-1}^{p-1} + C_{m-p-2}^{p-2})a^{m-2p} + \ldots$$

Ushbu tenglikda ma’lum,

$$C_n^k = C_{n-1}^k + C_{n-1}^{k-1}$$

formulani qo‘llab, isbotlanishi lozim bo‘lgan (4) tenglikka kelamiz.

7. Quyidagi tenglikni isbotlang.

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11} = \frac{1}{2}.$$

Isboti.

$$S = \cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11},$$

$$T = \sin\frac{\pi}{11} + \sin\frac{3\pi}{11} + \sin\frac{5\pi}{11} + \sin\frac{7\pi}{11} + \sin\frac{9\pi}{11},$$

deb belgilash kiritib, $V = S + Ti$ ifodani qaraylik.

$\alpha = \cos\frac{\pi}{11} + i\sin\frac{\pi}{11}$ deb olsak, u holda

$$V = S + Ti = \alpha + \alpha^3 + \alpha^5 + \alpha^7 + \alpha^9 = \frac{\alpha^{11} - \alpha}{\alpha^2 - 1} =$$

$$= \frac{\alpha(\alpha^{10} - 1)}{\alpha(\alpha - \alpha^{-1})} = \frac{\alpha^{10} - 1}{\alpha - \alpha^{-1}} = \frac{\alpha^5(\alpha^5 - \alpha^{-5})}{\alpha - \alpha^{-1}}.$$

Ushbu ifodada

$$\alpha^5 = \cos\frac{5\pi}{11} + i\sin\frac{5\pi}{11},\quad \alpha^{-5} = \cos\frac{5\pi}{11} - i\sin\frac{5\pi}{11}$$

tenglilardan foydalansak, u holda

$$V = S + Ti = \frac{\alpha^5(\alpha^5 - \alpha^{-5})}{\alpha - \alpha^{-1}} = \frac{(\cos\frac{5\pi}{11} + i\sin\frac{5\pi}{11})\cdot 2i\sin\frac{5\pi}{11}}{2i\sin\frac{\pi}{11}} =$$

$$= \frac{\cos\frac{5\pi}{11}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} + i\frac{\sin^2\frac{5\pi}{11}}{\sin\frac{\pi}{11}} = \frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}} + i\frac{\sin^2\frac{5\pi}{11}}{\sin\frac{\pi}{11}}$$

Shunday qilib,

$$S=\frac{\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{\sin\frac{\pi}{11}}=\frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{\sin(\pi-\frac{\pi}{11})}{2\sin\frac{\pi}{11}}=\frac{1}{2}.$$

Yuqoridagi isbotlanishi lozim bo'lgan tenglik isbotlandi.

8. Quyidagi ayniyatni isbotlang:

$$C_n^0+C_n^3+C_n^6+\ldots=\frac{1}{3}\left(2^n+2\cos\frac{n\pi}{3}\right).$$

Yechish. Ushbu ayniyatni qaraylik:

$$(1+x)^n=C_n^0+C_n^1x+C_n^2x^2+C_n^3x^3+\ldots+C_n^{n-1}x^{n-1}+C_n^nx^n.$$

Bu ayniyatga $x=1,\ \varepsilon,\ \varepsilon^2$, bunda $\varepsilon^2+\varepsilon+1=0$, qiymatlarni navbat bilan qo'yamiz:

$$2^n=C_n^0+C_n^1+C_n^2+C_n^3+\ldots,$$

$$(1+\varepsilon)^n=C_n^0+C_n^1\varepsilon+C_n^2\varepsilon^2+C_n^3\varepsilon^3+\ldots,$$

$$(1+\varepsilon^2)^n=C_n^0+C_n^1\varepsilon^2+C_n^3\varepsilon^4+C_n^3\varepsilon^6+\ldots$$

Lekin, agar k soni 3 ga bo'linmasa, u holda $1+\varepsilon^\kappa+\varepsilon^{2\kappa}=0$ va agar k soni 3 ga bo'linsa, $1+\varepsilon^\kappa+\varepsilon^{2\kappa}=3$.

Demak,

$$2^n+(1+\varepsilon)^n+(1+\varepsilon^2)^n=3\{C_n^0+C_n^3+C_n^6+\ldots\}.$$

Ma'lumki, ε uchun

$$\varepsilon=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$$

qiymatni qabul qilishi mumkin, u holda

$$1+\varepsilon=-\varepsilon^2=-\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3},$$

$$1+\varepsilon^2=-\varepsilon=-\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}=\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}.$$

Bundan esa

$$2^n + (1+\varepsilon)^n + (1+\varepsilon^2)^n = 2^n + 2\cos\frac{n\pi}{3}.$$

Demak,

$$C_n^0 + C_n^3 + C_n^6 + \ldots = \frac{1}{3}\left(2^n + 2\cos\frac{n\pi}{3}\right).$$

9. Quyidagi tenglikni isbotlang.

$$2^{2m} cos^{2m}\varphi = \sum_{\kappa=0}^{m-1} 2C_{2m}^{\kappa} cos 2(m-\kappa)\varphi + C_{2m}^{m}.$$

Isboti. Ma'lumki, $\cos\varphi = \dfrac{(\cos\varphi + i\sin\varphi) - (\cos\varphi - i\sin\varphi)}{2}$.

$cos\varphi + isin\varphi = z$ bo'lsin. Unda $cos\varphi - isin\varphi = z^{-1}$,

$$cos^{2m}\varphi = \left(\frac{z + z^{-1}}{2}\right)^{2m} = \frac{1}{2^{2m}}\sum_{\kappa=0}^{2m} C_{2m}^{\kappa} z^{-\kappa} z^{2m-\kappa} \Rightarrow$$

$$2^{2m} cos^{2m}\varphi = \sum_{\kappa=0}^{m-1} C_{2m}^{\kappa} z^{2(m-\kappa)} + C_{2m}^{m} + \sum_{\kappa=m+1}^{2m} C_{2m}^{\kappa} z^{2(m-\kappa)}.$$

Ikkinchi yig'indida $m-k = -(m-k^1)$ almashtirish qilamiz, u holda:

$$\sum_{\kappa'=m-1`}^{0} C_{2m}^{2m-\kappa'} z^{-2(m-\kappa')} = \sum_{\kappa=0`}^{m-1} C_{2m}^{\kappa} z^{-2(m-\kappa)}.$$

Natijada ushbu tenglikni hosil qilamiz:

$$2^{2m} cos^{2m}\varphi = \sum_{\kappa=0}^{m-1} C_{2m}^{\kappa}\left(z^{2(m-\kappa)} + z^{-2(m-\kappa)}\right) + C_{2m}^{m}.$$

Biroq $z^{2(m-\kappa)} + z^{-2(m-\kappa)} = 2cos2(m-k)$, u holda

$$2^{2m}\cos^{2m}\varphi = \sum_{\kappa=0}^{m-1} 2C_{2m}^{\kappa}\cos 2(m-\kappa)\varphi + C_{2m}^{m}.$$

10. Yig'indini toping.

$$S = \sin^2 x + \sin^2 3x + \sin^2 5x + \ldots + \sin^2(2n-1)x.$$

Yechish. Elementar matematika kursidan ma'lum bo'lgan

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

formuladan foydalanamiz. U holda

$$S = \frac{1-\cos 2x}{2} + \frac{1-\cos 6x}{2} + \frac{1-\cos 10x}{2} + \ldots + \frac{1-\cos[4n-2]x}{2} =$$

$$= \frac{n}{2} - \frac{1}{2}\left[\cos 2x + \cos 6x + \cos 10x + \ldots + \cos(4n-2)x\right]$$

Endi

$$s = \cos 2x + \cos 6x + \cos 10x + \ldots + \cos(4n-2)x,$$

$$t = \sin 2x + \cos 6x + \cos 10x + \ldots + \cos(4n-2)x$$

yig'indilarni hisoblaylik.

$$\alpha = \cos 2x + i\sin 2x$$

deb belgilash kiritaylik, u holda

$$s + it = \alpha + \alpha^3 + \ldots + \alpha^{2n-1} = \frac{\alpha^{2n+1} - \alpha}{\alpha^2 - 1} = \frac{\alpha^n(\alpha^n - \alpha^{-n})}{\alpha - \alpha^{-1}} =$$

$$= \frac{(\cos 2nx + i\sin 2nx)\cdot 2i\sin 2nx}{2i\sin 2x} = \frac{\cos 2nx\sin 2nx}{\sin 2x} + i\frac{\sin^2 2nx}{\sin 2x}.$$

Demak,

$$S = \frac{n}{2} - \frac{1}{2}\cdot\frac{\cos 2nx\sin 2nx}{\sin 2x} = \frac{n}{2} - \frac{\sin 4nx}{4\sin 2x}.$$

2. Kompleks sonlar yordamida yig'indilarni hisoblash.

1. Quyidagi yig'indini hisoblang.

$$S = 1 + a\cos x + a^2\cos 2x + a^3\cos 3x + \ldots + a^k\cos kx\ .$$

Yechish.

$$T = a\sin x + a^2\sin 2x + a^3\sin 3x + \ldots + a^k\sin kx$$

deb belgilash kiritaylik. U holda

$$S + Ti = 1 + a(\cos x + i\sin x) + a^2(\cos 2x + i\sin 2x) +$$

$$+ a^3(\cos 3x + i\sin 3x) + \ldots + a^k(\cos kx + i\sin kx)$$

bo'ladi. $\alpha = \cos x + i\sin x$ deb olsak, u holda

$$S+Ti=1+a\alpha+a^2\alpha^2+a^3\alpha^3+...+a^k\alpha^k=\frac{a^{k+1}\alpha^{k+1}-1}{a\alpha-1}.$$

S hosil qilingan yig'indining haqiqiy qismiga teng, shu sababli

$$S+Ti=\frac{a^{k+1}\alpha^{k+1}-1}{a\alpha-1}\cdot\frac{a\alpha^{-1}-1}{a\alpha^{-1}-1}=\frac{a^{k+2}\alpha^{k}-a^{k+1}\alpha^{k+1}+a\alpha^{-1}+1}{a^2-a(\alpha+\alpha^{-1})+1}.$$

Bu yerdan

$$S=\frac{a^{k+2}\cos kx-a^{k+1}\cos(k+1)x-a\cos x+1}{a^2-2a\cos x+1}.$$

2. Quyidagi tenglikni isbotlang.

$$\sin x+\sin 2x+\sin 3x+...+\sin nx=\frac{\sin\frac{n+1}{2}x\sin\frac{nx}{2}}{\sin\frac{x}{2}}.$$

Yechish.

$$T=\sin x+\sin 2x+\sin 3x+...+\sin nx,$$

$$S=\cos x+\cos 2x+\cos 3x+...+\cos nx,$$

$$\alpha=\cos\frac{x}{2}+i\sin\frac{x}{2},$$

bo'lsin. U holda

$$S+Ti=\alpha^2+\alpha^4+\alpha^6+...+\alpha^{2n}=\alpha^2\frac{\alpha^{2n}-1}{\alpha^2-1}=$$

$$=\alpha^2\frac{\alpha^n(\alpha^n-\alpha^{-n})}{\alpha(\alpha-\alpha^{-1})}=(\cos\frac{n+1}{2}x+i\sin\frac{n+1}{2}x)\frac{\sin\frac{n}{2}x}{\sin\frac{x}{2}}.$$

Bu yerdan quyidagi tengliklarni hosil qilamiz:

$$T=\sin x+\sin 2x+\sin 3x+...+\sin nx=\frac{\sin\frac{n+1}{2}x\sin\frac{nx}{2}}{\sin\frac{x}{2}},$$

$$S = \cos x + \cos 2x + \cos 3x + \ldots + \cos nx = \frac{\cos\frac{n+1}{2}x \sin\frac{nx}{2}}{\sin\frac{x}{2}}.$$

3. Quyidagi limitni hisoblang.

$$\lim_{n\to\infty}(1 + \frac{1}{2}\cos x + \frac{1}{4}\cos 2x + \ldots + \frac{1}{2^n}\cos nx)$$

Yechish. 1-misolda $a = \frac{1}{2}$ deb olib, undan foydalanamiz.

$$\lim_{n\to\infty}\left(1 + \frac{1}{2}\cos x + \frac{1}{4}\cos 2x + \ldots + \frac{1}{2^n}\cos nx\right) =$$

$$= \lim_{n\to\infty}\left(\frac{\left(\frac{1}{2}\right)^{n+2}\cos nx - \left(\frac{1}{2}\right)^{n+1}\cos(n+1)x - \frac{1}{2}\cos x + 1}{\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)\cos x + 1}\right) =$$

$$= \frac{-\frac{1}{2}\cos x + 1}{\frac{1}{4} - \cos x + 1} = \frac{2(2 + \cos x)}{5 - 4\cos x}$$

4. Quyidagi yig'indilarni hisoblang.

$$S = \cos a - \cos(a + h) + \cos(a + 2h) - \ldots + (-1)^{n-1}\cos(a + (n-1)h),$$

$$T = \sin a - \sin(a + h) + \sin(a + 2h) - \ldots + (-1)^{n-1}\sin(a + (n-1)h).$$

Yechish. Quyidagi belgilashlar kiritaylik.

$$\alpha = \cos a + i\sin a, \quad \beta = \cosh + i\sinh .$$

U holda

$$\alpha \cdot \beta = \cos(a + h) + i\sin(a + h),$$

$$\alpha \cdot \beta^2 = \cos(a + 2h) + i\sin(a + 2h),$$

....

$$\alpha \cdot \beta^{n-1} = \cos(a + (n-1)h + i\sin(a + (n-1)h).$$

Ushbu tengliklardan

$$V = S + iT = \alpha - \alpha\beta + \alpha\beta^2 - ... + (-1)^{n-1}\alpha\beta^{n-1},$$

uni hisoblaylik. Bu hisoblashda geometrik prosressiyaning hadlari yig'indisi formulasidan foydalanamiz.

$$V = \alpha(1 - \beta + \beta^2 - \beta^3 + ...(-)^{n-1}\beta^{n-1}) = \alpha \cdot \frac{(-1)^n\beta^n - 1}{-\beta - 1} = \alpha \cdot \frac{(-1)^{n+1}\beta^n + 1}{\beta + 1}.$$

$n -$ juft son bo'lsin. U holda

$$V = \alpha \cdot \frac{-\beta^n + 1}{\beta + 1} = \alpha \cdot \frac{(-\beta^n + 1)(\beta - 1)}{\beta^2 - 1} = \alpha \cdot \frac{\beta^{\frac{n}{2}}(\beta^{\frac{n}{2}} - \beta^{-\frac{n}{2}})(1 - \beta)}{\beta(\beta - \beta^{-1})} =$$

$$= \alpha \cdot \frac{(\beta^{\frac{n}{2}-1} - \beta^{\frac{n}{2}})(\beta^{\frac{n}{2}} - \beta^{-\frac{n}{2}})}{\beta - \beta^{-1}} = \frac{(\alpha\beta^{\frac{n}{2}-1} - \alpha\beta^{\frac{n}{2}})(\beta^{\frac{n}{2}} - \beta^{-\frac{n}{2}})}{\beta - \beta^{-1}}$$

Endi α va β larni yuqorida belgilaganimizga asosan V ni ifodasiga qo'yaylik:

$$V = \frac{\left\{\left[\cos\left(a + \left(\frac{n}{2} - 1\right)h\right) + i\sin\left(a + \left(\frac{n}{2} - 1\right)h\right)\right] - \left[\cos\left(a + \frac{n}{2}h\right) + i\sin\left(a + \frac{n}{2}h\right)\right]\right\} \cdot \sin\frac{n}{2}h}{\sin h}$$

Bu yerdan

$$S = \frac{\left[\cos\left(a + \left(\frac{n}{2} - 1\right)h\right) - \cos\left(a + \frac{n}{2}h\right)\right]\sin\frac{n}{2}h}{\sin h} =$$

$$= \frac{-2\sin\left(a + \frac{n-1}{2}h\right)\sin\left(-\frac{h}{2}\right)\sin\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}} = \frac{\sin\left(a + \frac{n-1}{2}h\right)\sin\frac{n}{2}h}{\cos\frac{h}{2}}.$$

$$T = \frac{-\left[\sin\left(a + \left(\frac{n}{2} - 1\right)h\right) - \sin\left(a + \frac{n}{2}h\right)\right]\sin\frac{n}{2}h}{\sin h} =$$

$$= \frac{2\cos\left(a + \frac{n-1}{2}h\right)\sin\frac{h}{2} \cdot \sin\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}} = \frac{\cos\left(a + \frac{n-1}{2}h\right)\sin\frac{n}{2}h}{\cos\frac{h}{2}}.$$

$n-$ toq son bo'lsin. U holda

$$V=\alpha\cdot\frac{-\beta^{n}-1}{-\beta-1}=\alpha\cdot\frac{(\beta^{n}+1)(\beta-1)}{\beta^{2}-1}=\alpha\cdot\frac{\beta^{\frac{n}{2}}(\beta^{\frac{n}{2}}+\beta^{-\frac{n}{2}})(\beta-1)}{\beta(\beta-\beta^{-1})}=$$

$$=\alpha\cdot\frac{(\beta^{\frac{n}{2}}-\beta^{\frac{n}{2}-1})(\beta^{\frac{n}{2}}+\beta^{-\frac{n}{2}})}{\beta-\beta^{-1}}=\frac{(\alpha\beta^{\frac{n}{2}}-\alpha\beta^{\frac{n}{2}-1})(\beta^{\frac{n}{2}}+\beta^{-\frac{n}{2}})}{\beta-\beta^{-1}}$$

α va β larni V ning ifodasiga qo'yaylik:

$$V=\frac{\left\{\left[\cos\left(a+\frac{n}{2}h\right)+i\sin\left(a+\frac{n}{2}h\right)\right]-\left[\cos\left(a+\frac{n-2}{2}h\right)+i\sin\left(a+\frac{n-2}{2}h\right)\right]\right\}\cdot\cos\frac{n}{2}h}{i\sin h}$$

Bu yerdan

$$S=\frac{\left[\sin\left(a+\frac{n}{2}h\right)-\sin\left(a+\frac{n-2}{2}h\right)\right]\cos\frac{n}{2}h}{\sin h}=$$

$$=\frac{2\cos\left(a+\frac{n-1}{2}h\right)\sin\frac{h}{2}\cos\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}}=\frac{\cos(a+\frac{n-1}{2}h)\cos\frac{n}{2}h}{\cos\frac{h}{2}}.$$

$$T=\frac{-\left[\cos\left(a+\frac{n}{2}h\right)-\cos\left(a+\frac{n-2}{2}h\right)\right]\cos\frac{n}{2}h}{\sin h}=$$

$$=\frac{2\sin\left(a+\frac{n-1}{2}h\right)\sin\frac{h}{2}\cdot\cos\frac{n}{2}h}{2\sin\frac{h}{2}\cos\frac{h}{2}}=\frac{\sin\left(a+\frac{n-1}{2}h\right)\cos\frac{n}{2}h}{\cos\frac{h}{2}}.$$

Shundy qilib,

$$S=\frac{\sin\left(a+\frac{n-1}{2}h\right)\sin\frac{n}{2}h}{\cos\frac{h}{2}},\qquad n\text{–juft bo'lsa,}$$

$$S = \frac{\cos\left(a + \frac{n-1}{2}h\right)\cos\frac{n}{2}h}{\cos\frac{h}{2}}, \qquad n - \text{toq bo‘lsa},$$

$$T = \frac{\cos(a + \frac{n-1}{2}h)\sin\frac{n}{2}h}{\cos\frac{h}{2}}, \qquad n - \text{juft bo‘lsa},$$

$$T = \frac{\sin\left(a + \frac{n-1}{2}h\right)\cos\frac{n}{2}h}{\cos\frac{h}{2}}, \qquad n - \text{toq bo‘lsa}.$$

5. Agar x absolyut qiymati jihatidan birdan kichik bo‘lsa, u holda

a) $S = \cos a + x\cos(a+b) + x^2\cos(a+2b) + ... + x^n\cos(a+nb) + ...$

b) $T = \sin a + x\sin(a+b) + x^2\sin(a+2b) + ... + x^n\sin(a+nb) + ...$

qatorlar yaqinlashuvchi ekanligini va ularning yig‘indisi mos ravishda

$$\frac{\cos a - x\cos(a-b)}{1 - 2x\cos b + x^2}, \qquad \frac{\sin a - x\sin(a-b)}{1 - 2x\cos b + x^2}$$

teng ekanligini isbotlang.

Isboti.

$$S_k = \cos a + x\cos(a+b) + x^2\cos(a+2b) + ... + x^k\cos(a+kb),$$

$$T_k = \sin a + x\sin(a+b) + x^2\sin(a+2b) + ... + x^k\sin(a+kb),$$

deb belgilash kiritaylik. U holda

$$S_k + iT_k = (\cos a + i\sin a) + x[\cos(a+b) + i\sin(a+b)] +$$
$$+ x^2[\cos(a+2b) + i\sin(a+2b)] + ... + x^k[\cos(a+kb) + i\sin(a+kb)]$$

bo‘ladi.

$$\alpha = \cos a + i\sin a, \qquad \beta = \cos b + i\sin b$$

deb olsak, u holda

$$S_k + iT_k = \alpha(1 + x\beta + x^2\beta^2 + x^3\beta^3 + ... + x^k\beta^k) =$$

$$= \frac{\alpha(x^{k+1}\beta^{k+1} - 1)}{x\beta - 1} \cdot \frac{x\beta^{-1} - 1}{x\beta^{-1} - 1} = \frac{\alpha(x^{k+2}\beta^k - x^{k+1}\beta^{k+1} + x\beta^{-1} + 1)}{x^2 - x(\beta + \beta^{-1}) + 1}$$

Demak,

$$S_k = \frac{x^{k+2}\cos(a + kb) - x^{k+1}\cos[a + (k+1)b] - x\cos(a - b) + \cos a}{x^2 - 2x\cos b + 1},$$

$$T_k = \frac{x^{k+2}\sin(a + kb) - x^{k+1}\sin[a + (k+1)b] - x\sin(a - b) + \sin a}{x^2 - 2x\cos b + 1}.$$

Ushbu yig'indilarda $k \to \infty$ da limitga o'tsak, u holda $|x| < 1$ bo'lgani uchun

$$\lim_{k \to \infty} |x|^k = 0$$

bo'ladi. Shunday qilib,

$$S = \lim_{k \to \infty} S_k = \frac{\cos a - x\cos(a - b)}{1 - 2x\cos b + x^2},$$

$$T = \lim_{k \to \infty} T_k = \frac{\sin a - x\sin(a - b)}{1 - 2x\cos b + x^2}.$$

6. Quyidagi tengliklarni isbotlang.

$$\cos^2 x + \cos^2 2x + \cos^2 3x + ... + \cos^2 nx = \frac{n}{2} + \frac{\cos(n+1)x\sin nx}{2\sin x};$$

$$\sin^2 x + \sin^2 2x + \sin^2 3x + ... + \sin^2 nx = \frac{n}{2} - \frac{\cos(n+1)x\sin nx}{2\sin x}.$$

Isboti. 1-banddagi 10–misolga o'xshash $\cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$ va $\sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$ formulalardan foydalanamiz. U holda

$$\cos^2 x + \cos^2 2x + ... + \cos^2 nx = \frac{1 + \cos 2x}{2} + \frac{1 + \cos 4x}{2} + ... + \frac{1 + \cos 2nx}{2} =$$

$$= \frac{n}{2} + \frac{1}{2}[\cos 2x + \cos 4x + ... + \cos 2nx];$$

$$\sin^2 x + \sin^2 2x + ... + \sin^2 nx = \frac{1 - \cos 2x}{2} + \frac{1 - \cos 4x}{2} + ... + \frac{1 - \cos 2nx}{2} =$$

$$= \frac{n}{2} - \frac{1}{2}[\cos 2x + \cos 4x + ... + \cos 2nx].$$

Endi

$$s = \cos 2x + \cos 4x + \ldots + \cos 2nx, \quad t = \sin 2x + \sin 4x + \ldots + \sin 2nx$$

yig'indilarni hisoblaylik.

$$s + it = (\cos 2x + i\sin 2x) + (\cos 4x + i\sin 4x) + \ldots + (\cos 2nx + i\sin 2nx) = \alpha^2 + \alpha^4 +$$

$$\ldots + \alpha^{2n} = \frac{\alpha^{2n+2} - \alpha^2}{\alpha^2 - 1} = \frac{\alpha^{n+1}(\alpha^n - \alpha^{-n})}{\alpha - \alpha^{-1}} = \frac{[\cos(n+1)x + i\sin(n+1)x]\cdot 2i\sin nx}{2i\sin x} =$$

$$= \frac{\cos(n+1)x\sin nx}{\sin x} + i\frac{\sin(n+1)x\sin nx}{\sin x}.$$

Ushbu tenglikdan esa isbotlanishi lozim bo'lgan tenglik kelib chiqadi:

$$\cos^2 x + \cos^2 2x + \ldots + \cos^2 nx = \frac{n}{2} + \frac{1}{2}[\cos 2x + \cos 4x + \ldots + \cos 2nx] =$$

$$= \frac{n}{2} + \frac{\cos(n+1)x\sin nx}{2\sin x};$$

$$\sin^2 x + \sin^2 2x + \ldots + \sin^2 nx = \frac{n}{2} - \frac{1}{2}[\cos 2x + \cos 4x + \ldots + \cos 2nx] =$$

$$= \frac{n}{2} - \frac{\cos(n+1)x\sin nx}{2\sin x}.$$

7. Yig'indilarni toping.

$$S = \cos^3 x + \cos^3 2x + \ldots. + \cos^3 nx, \quad T = \sin^3 x + \sin^3 2x + \ldots. + \sin^3 nx.$$

Yechish. Yuqorida isbotlangan

$$\cos^3\alpha = \frac{3\cos\alpha - \cos 3\alpha}{4}, \quad \sin^3\alpha = \frac{3\sin\alpha - \sin 3\alpha}{4}$$

tengliklardan foydalanamiz. U holda

$$S = \frac{1}{4}[3(\cos x + \cos 2x + \ldots + \cos nx) + (\cos 3x + \cos 6x + \ldots + \cos 3nx)];$$

$$T = \frac{1}{4}[3(\sin x + \sin 2x + \ldots + \sin nx) - (\sin 3x + \sin 6x + \ldots + \sin 3nx)].$$

Quyidagicha belgilashlar kiritaylik:

$$\alpha = \cos\frac{x}{2} + i\sin\frac{x}{2}, \quad \beta = \cos\frac{3x}{2} + i\sin\frac{3x}{2}.$$

U holda

$$S_1 + iT_1 = (\cos x + \sin x) + (\cos 2x + i\sin 2x) + \ldots + (\cos nx + i\sin nx) = \alpha^2 + \alpha^4 +$$

$$+ \ldots + \alpha^{2n} = \frac{\alpha^{2n+2} - \alpha^2}{\alpha^2 - 1} = \frac{\alpha^{n+1}(\alpha^n - \alpha^{-n})}{\alpha - \alpha^{-1}} =$$

$$= \frac{(\cos\frac{n+1}{2}x + i\sin\frac{n+1}{2}x)\cdot 2i\sin\frac{n}{2}x}{2i\sin\frac{x}{2}} = \frac{\cos\frac{n+1}{2}x\sin\frac{n}{2}x}{\sin\frac{x}{2}} + i\frac{\sin\frac{n+1}{2}x\sin\frac{n}{2}x}{\sin\frac{x}{2}},$$

$$S_2 + iT_2 = (\cos 3x + \sin 3x) + (\cos 6x + i\sin 6x) + \ldots + (\cos 3nx + i\sin 3nx) =$$

$$= \beta^2 + \beta^4 + \ldots + \beta^{2n} = \frac{\beta^{2n+2} - \beta^2}{\beta^2 - 1} = \frac{\beta^{n+1}(\beta^n - \beta^{-n})}{\beta - \beta^{-1}} =$$

$$= \frac{(\cos\frac{3(n+1)}{2}x + i\sin\frac{3(n+1)}{2}x)\cdot 2i\sin\frac{3n}{2}x}{2i\sin\frac{3x}{2}} =$$

$$= \frac{\cos\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{\sin\frac{3x}{2}} + i\frac{\sin\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{\sin\frac{3x}{2}},$$

Demak,

$$S = \frac{3\cos\frac{n+1}{2}x\sin\frac{n}{2}x}{4\sin\frac{x}{2}} + \frac{\cos\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{4\sin\frac{3x}{2}};$$

$$T = \frac{3\sin\frac{n+1}{2}x\sin\frac{n}{2}x}{4\sin\frac{x}{2}} - \frac{\sin\frac{3(n+1)}{2}x\sin\frac{3n}{2}x}{4\sin\frac{3x}{2}}.$$

8. Quyidagi tenglikni isbotlang:

$$sin\frac{\pi}{2m}\cdot sin\frac{2\pi}{2m}\cdot\ldots\cdot sin\frac{(m-1)\pi}{2m} = \frac{\sqrt{m}}{2^{m-1}}.$$

Yechish. Dastlab ishni $x^n - 1$ ni ko‘paytuvchilarga ajratishdan boshlaymiz.

$n = 2m$ bo‘lsa, u holda $x^n - 1 = 0$ tenglama ikkita haqiqiy ildizga: 1, -1 va $2m - 2$ kompleks ildizga ega bo‘ladi.

Ko‘rsatish mumkinki, $\varepsilon_\kappa = cos\frac{2\pi\kappa}{2m} + isin\frac{2\pi\kappa}{2m}$ kompleks sonning qo‘shmasi

$$\varepsilon_{2m-\kappa} = cos\frac{2(2m-\kappa)\pi}{2m} + isin\frac{2(2m-\kappa)\pi}{2m}$$

bo‘ladi. Bundan quyidagi hosil bo‘ladi:

$$x^{2m} - 1 = (x^2 - 1)(x - \varepsilon_1)(x - \overline{\varepsilon_1})(x - \varepsilon_2)(x - \overline{\varepsilon_2})...(x - \varepsilon_{m-1})(x - \overline{\varepsilon_{m-1}})$$

$$x^{2m} - 1 = (x^2 - 1)(x^2 - (\varepsilon_1 + \overline{\varepsilon_1})x + 1)...(x^2 - (\varepsilon_{m-1} + \overline{\varepsilon_{m-1}})x + 1)$$

$$x^{2m} - 1 = (x^2 - 1)\prod_{\kappa=1}^{m-1}\left(x^2 - 2xcos\frac{\kappa\pi}{m} + 1\right).$$

Agar $n = 2m + 1$ bo‘lsa, u holda yuqoridagi kabi quyidagi tenglikni hosil qilamiz:

$$x^{2m+1} - 1 = (x - 1)\prod_{\kappa=1}^{m}\left(x^2 - 2xcos\frac{2\kappa\pi}{2m+1} + 1\right).$$

Hosil bo‘lgan tenglikni ushbu ko‘rinishda yozamiz:

$$\frac{x^{2m} - 1}{x^2 - 1} = \prod_{\kappa=1}^{m-1}\left(x^2 - 2x\cos\frac{\kappa\pi}{m} + 1\right).$$

Bu ifodada $x = 1$ desak, quyidagi tenglikni hosil qilamiz:

$$m = 2^{m-1}\prod_{\kappa=1}^{m-1}\left(1 - cos\frac{\kappa\pi}{m}\right) \text{ yoki } m = 2^{2(m-1)}\prod_{\kappa=1}^{m-1}sin^2\frac{\kappa\pi}{2m},$$

va nihoyat,

$$\frac{\sqrt{m}}{2^{m-1}} = \prod_{\kappa=1}^{m-1}sin\frac{\kappa\pi}{2m}.$$

9. Quyidagi tenglamani yeching

$$cos\varphi + C_n^1 cos(\varphi + \alpha)x + ... + cos(\varphi + n\alpha)x^n = 0$$

Yechish. Quyidagi yig‘indilarni olaylik:

$$S = \cos\varphi + C_n^1 \cos(\varphi + \alpha)x + \ldots + \cos(\varphi + n\alpha)x^n,$$

$$T = \sin\varphi + C_n^1 \sin(\varphi + \alpha)x + \ldots + \sin(\varphi + n\alpha)x^n.$$

Unda

$$S + Ti = \mu(1 + \lambda x)^n \text{ va } S - Ti = \overline{\mu}(1 + \overline{\lambda} x)^n,$$

bu yerda $\lambda = \cos\alpha + i\sin\alpha$, $\mu = \cos\varphi + i\sin\varphi$.

Bu tengliklarni hadlab qo'shib, quyidagini hosil qilamiz:

$$2S = \mu(1 + \lambda x)^n + \overline{\mu}(1 + \overline{\lambda} x)^n.$$

Natijada berilgan tenglama quyidagi ko'rinishga keladi:

$$\mu(1 + \lambda x)^n + \overline{\mu}(1 + \overline{\lambda} x)^n = 0.$$

Ushbu tenglamani yechib, noma'lumni topamiz:

$$x_k = -\frac{\sin\dfrac{(k+1)\pi - 2\varphi}{2n}}{\sin\dfrac{(k+1)\pi - 2\varphi - 2n\alpha}{2n}}; \quad k = 0,1,2,\ldots n-1.$$

10. Ushbu $\sin^n nx$ va $\cos^n nx$ (n – natural son) darajalarni x ga karrali burchaklar kosinus va sinuslarining birinchi darajali ko'phadi ko'rinishda ifodalash mumkinligini isbotlang.

Yechish.

$z = \cos x + i\sin x$ deb olaylik, unda $\overline{z} = \cos x - i\sin x$ bo'ladi. Natijada esa quyidagi formulalarni hosil qilamiz:

$$\cos x = \frac{z + \overline{z}}{2}, \ \sin x = \frac{z - \overline{z}}{2i}, \ \cos^n x = \frac{1}{2^n}(z + \overline{z})^n = \frac{1}{2^n}\sum_{k=0}^{n} C_n^k z^{n-k}\overline{z}^k,$$

$$\sin^n x = \frac{1}{(2i)^n}(z - \overline{z})^n = \frac{1}{(2i)^n}\sum_{k=0}^{n} (-1)^k C_n^k z^{n-k}\overline{z}^k.$$

Keyin bir xil binomial koeffisientli hadlarni guruhlab, $z^k \overline{z} = 1$ umumiy ko'paytuvchini qavsdan tashqariga chiqarib va Muavr formulasidan foydalansak, mos ravishda quyidagi tengliklarni hosil qilamiz:

agar $n = 2\lambda + 1$ bo'lsa, u holda

$$sin^n x = (-1)^{\frac{n-1}{2}} \left(\frac{1}{2}\right)^{n-1} \sum_{k=0}^{\lambda} (-1)^k C_n^k sin(n-2k)x\,,$$

agar $n = 2\lambda$ bo'lsa, u holda

$$sin^n x = (-1)^{\frac{n}{2}} \left(\frac{1}{2}\right)^{n-1} \left[\sum_{k=0}^{\lambda-1} (-1)^k C_n^k cos(n-2k)x + (-1)^{\lambda} \frac{1}{2} C_n^{\lambda}\right],$$

agar $n = 2\lambda + 1$ bo'lsa, u holda $cos^n x = \left(\frac{1}{2}\right)^{n-1} \sum_{k=0}^{\lambda} C_n^k cos(n-2k)x$,

agar $n = 2\lambda$ bo'lsa, u holda $cos^n x = \left(\frac{1}{2}\right)^{n-1} \left[\sum_{k=0}^{\lambda-1} C_n^k cos(n-2k)x + \frac{1}{2} C_n^{\lambda}\right].$

XULOSA

O'ylaymizki, ba'zi hisoblash murakkab bo'lgan yig'indi va ko'paytmalarning qiymatlarini kompleks analiz metodlaridan foydalangan holda hisoblash va oson hisoblashga qulaylik tug'diradigan yo'llarini topish masalasining o'rganilishi kompleks sonlar nazariyasining zamonaviy kontseptsiyalarini osonroq tushunishga yordam beradi va shuningdek, kompleks analizning tatbiqlari bobining rivojlanishiga asos bo'ladi hamda haqiqiy analizning ba'zi masalalarini hal qilish usullarini yanada ko'paytiradi.

Ushbu maqolada kompleks sonlar ustida amallar bajarish orqali hosil bo'ladigan sonlarni haqiqiy qismini haqiqiy qismiga mavhum qismini mavhum qismiga tenglashtirib muhim natijalar erishildi.

Foydalanilgan adabiyotlar ro'yxati

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