NAMANGAN DAVLAT PEDAGOGIKA INSTITUTI
"TA'LIM VA TARAQQIYOT" ILMIY-USLUBIY
JURNALI 2026-YIL 1-SON

13.00.00. – PEDAGOGIKA FANLARI
JOURNAL.NAMSPI.UZ
ISSN: 2992-9008; UDK: 37

ELEMENTS OF VECTOR FIELDS: GRADIENT, DIVERGENCE, AND ROTOR EXPRESSION IN VARIOUS COORDINATE SYSTEMS AND THEIR APPLICATIONS

Nafasov Ganisher Abdurashidovich
Associate Professor of the Department of Mathematics, Doctor of Philosophy (PhD) in Pedagogical Sciences, Gulistan State University
E-mail: *gnafasov87@gmail.com*
Salimboyev Utkirjon Ne'matjon ogli
Master's student in Mathematics at Gulistan State University
Email: ***otkirsalimboyev@gmail.com***

Abstract. *This in the article vector area main differential operators — gradient , divergence and rotor Cartesian , cylindrical and spherical coordinates in systems expressions and their physicist applications studied . Every one operator's mathematician content , formulas and practical in matters application electrostatics, hydrodynamics and heat conductivity processes in the example of analysis was done.*

Key words : *vector area , gradient , divergence , rotor, coordinates system , electrostatics , hydrodynamics*

VEKTOR MAYDONI ELEMENTLARI: GRADIYENT, DIVERGENSIYA VA ROTORNING TURLI KOORDINATALAR SISTEMALARIDA IFODALANISHI VA ULARNING TATBIQLARI

Annotatsiya. *Mazkur maqolada vektor maydonining asosiy differensial operatorlari — gradiyent, divergensiya va rotorning Dekart, silindrik hamda sferik koordinatalar sistemalaridagi ifodalanishlari va ularning fizik tatbiqlari o'rganildi. Har bir operatorning matematik mazmuni, formulalari hamda amaliy masalalardagi qo'llanilishi elektrostatika, gidrodinamika va issiqlik o'tkazuvchanlik jarayonlari misolida tahlil qilindi.*

Kalit so'zlar: *vektor maydoni, gradiyent, divergensiya, rotor, koordinatalar sistemasi, elektrostatika, gidrodinamika.*

ЭЛЕМЕНТЫ ВЕКТОРНЫХ ПОЛЕЙ: ВЫРАЖЕНИЕ ГРАДИЕНТА, ДИВЕРГЕНЦИИ И РОТОРА В РАЗЛИЧНЫХ СИСТЕМАХ КООРДИНАТ И ИХ ПРИМЕНЕНИЕ

Аннотация. *В статье исследованы основные дифференциальные операторы векторного поля — градиент, дивергенция и ротор в декартовой, цилиндрической и сферической системах координат, а также их физические приложения. Математическое содержание, формулы и практическое применение каждого оператора проанализированы на примерах электростатики, гидродинамики и процессов теплопроводности.*

Ключевые слова: *векторное поле, градиент, дивергенция, ротор, система координат, электростатика, гидродинамика.*

INTRODUCTION

Vector fields modern physics almost all fundamental concepts in the sections Electricity and magnet fields , liquid and gases of the flow speed distribution , heat exchange , elasticity in theory deformation processes and quantum in mechanics probability flow such as many events vector fields through mathematician This is modeled . of the fields in space behavior quantitative in terms of in description gradient , divergence and like a rotor differential operators main role plays .

Gradient operator scalar of the fields in space the most fast change direction clearly if , divergence vector of the field source or absorption the characteristic , and the rotor of the field turnover character describes . This operators right chosen coordinates in systems expression of the matter to the symmetry suitable without calculation process noticeable at the level For example , spherical to symmetry has electrostatic fields spherical in coordinates analysis to do as a result equations one how many once becomes compact , this and analytical solutions to take opportunity increases .

Therefore , the vector area elements different coordinates in systems study not only theoretical importance have , maybe electrostatics , hydrodynamics , heat conductivity and electromagnetic fields in theory practical issues effective solution also important for scientific basis become service does .

METHODS

This research in the process vector analysis main differential operators different coordinates in systems expression and their physicist to processes implementation analysis to do for the purpose following scientific and methodological **from methods used :**

Theoretical analysis method. This method through vector analysis , mathematics physics and electromagnetic fields to the theory related classic and modern scientific literature systematic accordingly studied . In particular , gradient

, divergence and rotor operators Cartesian , cylindrical and spherical coordinates in systems general formulas identified , their mathematician essence analysis was done.

Comparison and generalization method . Different coordinates in systems operators structural differences and similar aspects comparative accordingly analysis was done . Every one coordinate system symmetry to the conditions compatibility evaluated , which physicist issues for which coordinates from the system use to the goal compatibility generalized .

Mathematician modeling Electrostatic , hydrodynamic **method** and heat permeability processes for typical physicist models selected and given to them relatively gradient , divergence and rotor operators was applied . As a result this real physics of operators processes descriptive functional importance practical examples through based on was given .

RESULTS

Gauss's theorem covers the concept of electric current. Let us consider a surface intersected by the lines of force of a uniform electric field of strength E (Fig. 1). If the electric field strength is perpendicular to the surface (Fig. 1a), then **the current of strength** Φ_E is defined as follows:

$$\Phi_E = EA$$

but forms an angle with it , then it crosses fewer lines of force. In this case, the flux of force passing through the surface is determined by the following formula: Θ

$$\Phi_E = EA_\perp = EA\cos\Theta$$

Here $A_\perp$ is the projection of the surface A onto a plane perpendicular to E. The surface A can be represented by a vector A directed perpendicular to its surface and proportional to the area, then the angle between E and A Θ and the flux of the current can be written as:

$$\Phi_E = E_\perp A = EA_\perp = EA\cos\Theta \quad (1)$$

where is $E_\perp = E\cos\Theta$ the constitutive element of E perpendicular to the surface (Fig. 1b) and, similarly, $A_\perp = A\cos\Theta$ is the projection of E perpendicular to the surface A (Fig. 1).

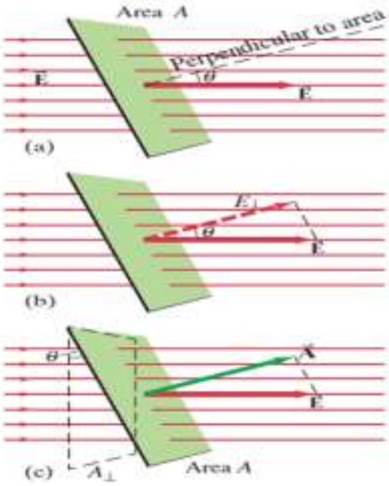


Figure 1. A uniform electric field EA passes through a surface: (a) – perpendicular to the lines of force, (b) – not perpendicular to the lines of force, (c) – projection onto a perpendicular plane.

The electric field can be explained based on the concept of lines of force. $A_\perp$ The number of lines of force passing through a unit area perpendicular to the direction of the field N is proportional to the electric field strength:

$$E \sim N / A_\perp$$

$$N \sim EA_\perp = \Phi_E \qquad (2)$$

that is, the flux of field strength passing through a surface is proportional to the number of lines of force crossing its surface.

Gauss's theorem applies to the total area of a non-homogeneous, non-flat space. We consider the space as shown in Figure 2. n We divide this surface into ta elements and $\Delta A_1, \Delta A_2, \ldots,$ denote their surfaces as etc. We divide the pieces in such a way that 1) each element is flat and 2) the electric field within the element can be considered uniform. Then the electric flux through the entire surface is given by:

$$\Phi_E = E_1 \Delta A_1 \cos\Theta_1 + E_2 \Delta A_2 \cos\Theta_1 + \ldots = \sum E \Delta A \cos\Theta = \sum E_\perp \Delta A$$

where is E_i the ΔA_i field strength on the element. $\Delta A_i \to 0$ At the boundary, it becomes integral over the entire surface and the equation is defined:

$$\Phi_E = \sum E_\perp \Delta A \sim Q_{encl}$$

The proportionality coefficient $1/\varepsilon_0$ corresponds to Coulomb's law, and we get:

NAMANGAN DAVLAT PEDAGOGIKA INSTITUTI
"TA'LIM VA TARAQQIYOT" ILMIY-USLUBIY
JURNALI 2026-YIL 1-SON

13.00.00. – PEDAGOGIKA FANLARI
JOURNAL.NAMSPI.UZ
ISSN: 2992-9008; UDK: 37

$$\sum E_{\perp} \Delta A = \frac{Q_{encl}}{\varepsilon_0}$$

where the sum ($\sum$) covers some closed surface and Q_{encl} is a particle on that closed surface. This expresses Gauss's theorem.

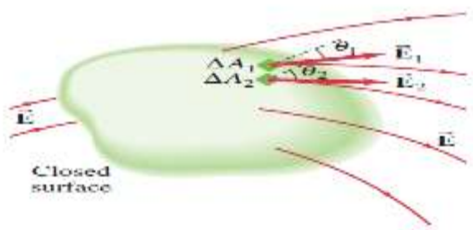


Figure 2. Determination of the force flow through a curved surface; $\Delta A_1, \Delta A_2, ...,$ etc. are the vector elements of the surface

1. Unlimited long charged right of the wire field strength .

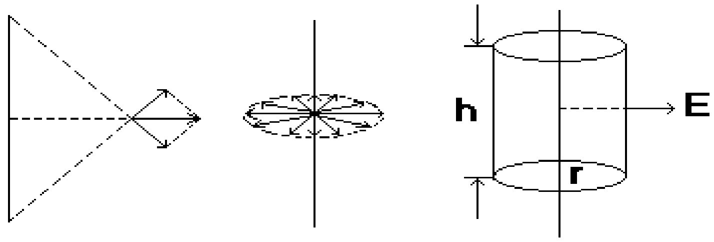


Figure 3

As seen from Figure 1 It is clear that E is connected to the wire . perpendicular . The wire cylindrical surface with we wrap . ρ - striped density (a meter long charge Gaussian to the theorem mainly :

$$\Phi = \frac{1}{\varepsilon_0} \sum_i^n q_i = \frac{\rho h}{\varepsilon_0} \quad (3)$$

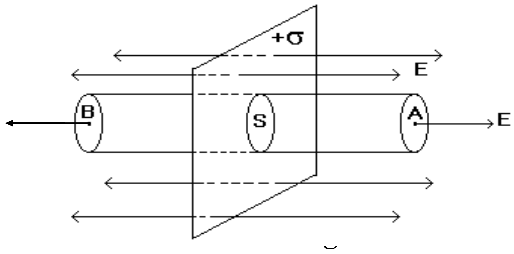
Here $\sum_i^n q_i = \rho h$ - cylinder inside charge . To the other $\Phi = ES = E \cdot 2\pi r h$, or ,

$\frac{\rho h}{\varepsilon_0} = E \cdot 2\pi r h$, from this $$E = \frac{\rho}{2\pi \varepsilon_0 r} \quad (4)$$

2. Charged endless of the plain field In this ***example*** raw E surface is perpendicular to the line . The force E at point A is we will find .

NAMANGAN DAVLAT PEDAGOGIKA INSTITUTI
"TA'LIM VA TARAQQIYOT" ILMIY-USLUBIY
JURNALI 2026-YIL 1-SON

13.00.00. – PEDAGOGIKA FANLARI
JOURNAL.NAMSPI.UZ
ISSN: 2992-9008; UDK: 37



To the surface perpendicular was cylinder surface Let's draw . Surface cylinder equal for two Gauss 's theorem mainly cylinder from the surface passing by stream

$$N = \frac{1}{\varepsilon_0}\sum_{i}^{n} q_i = \frac{\delta S}{\varepsilon_0} \text{ (5) to equal}$$

Here δ - surface in unity charge . OR $N = E \cdot 2S = \frac{\delta S}{\varepsilon_0}$.

So , $E = \frac{\delta}{2\varepsilon_0}$ (6) and it is from the surface was to the distance related it's not

.

Charged two parallel endless plain between field strength .

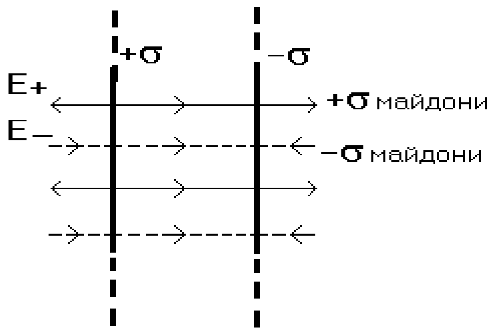


Figure 6

$|\sigma^+| = |\sigma^-|$ happened for $E_+ = E_- = \frac{\sigma}{2\sigma_0}$ (7) plains between $E = E_+ + E_-$ and

$E = \frac{\sigma}{\varepsilon_0}$ (8) will be .

From the plains outside $E = E_+ - E_-$ Therefore for outside E=0. So two on infinite parallel planes electricity area one gender they are only parallel planes between would be That's it .

Potential . electric field strength with potential between connection

From mechanics It is known that the forces potential in the field located body potential to energy has divided into squares forces this energy at the expense of work Electric in the field done the work potential energy difference to express as possible:

$$A = -(E_1 - E_2)\ , (9)$$

This equation with (9) if we compare , it is $_1 = \frac{qq_0}{4\pi\varepsilon_\varepsilon \varepsilon r_1}$ and eat $_2 = \frac{qq_0}{4\pi\varepsilon_\varepsilon \varepsilon r_2}$ is determined . Therefore , the mutual impact potential energy is $= \frac{qq_0}{4\pi\varepsilon_\varepsilon \varepsilon r}$. Electrostatic ***field potential*** φ and , tester what charge electrostatic field optional at the point potential energy is this charge to the amount ratio with identifiable physicist to size It is said , that is :

$$\varphi = \frac{E}{q} = \frac{q}{4\pi\varepsilon_0 \varepsilon r}\ . (10)$$

Potentially numerically unity positive charge on the field certain on point potential to the energy is equal to . Charges system harvest did field potential system to the composition entered every one charge separately harvest entered field potentials algebraic to the assembly is equal to .

If we potentials φ_1 and φ_2 to equal were , from each other Δ d=d $_2$ -d $_1$ m on the surface located two parallel plates given if , the area strength for

$$E = \frac{\varphi_2 - \varphi_1}{\Delta d} (11)$$

expression we can , this on the ground $\varphi_2 - \varphi_1 = U$ - plates between potentials difference or ***voltage*** The voltage in the XBS is called ***1V*** unit with is measured .

CONCLUSION

This in research vector of the fields main differential operators — gradient , divergence and rotor Cartesian , cylindrical and spherical coordinates in systems expressions systematic accordingly analysis was done . Obtained results this showed that this operators scalar and vector of the fields in space behavior fundamental mathematics in determining tool to be a physicist of processes the essence open in giving important role plays .

Research during coordinates system of the matter geometric to the symmetry suitable without choice calculations noticeable at the level simplify , analytical solutions to take opportunity increase and of the results accuracy to provide based on given Electrostatics , hydrodynamics and electromagnetic fields to the theory related examples gradient , divergence and real physics of rotor operators processes in modeling practical importance obvious manifestation reached .

Therefore , the vector analysis operators different coordinates in systems perfect mastery physicist research and engineering issues effective in solution important theoretical basis become service does .

References

1. Arfken , GB, Weber, HJ, & Harris, FE (2013). *Mathematical Methods for Physicists* (7th ed.). Academic Press .
2. Griffiths, DJ (2017). *Introduction to Electrodynamics* (4th ed.). Cambridge University Press.
3. Jackson, JD (1999). *Classical Electrodynamics* (3rd ed.). Wiley.
4. Kreiszig , E. (2011). *Advanced Engineering Mathematics* (10th ed.). Wiley.
5. Sadiku , MNO (2014). *Elements of Electromagnetics* (6th ed.). Oxford University Press.
6. Spiegel, M.R., Lipschutz , S., & Schiller, J. (2009). *Vector analysis* . McGraw-Hill.
7. Morse, PM, & Feshbach , H. (1953). *Methods of Theoretical Physics* . McGraw-Hill.
8. Stratton, JA (2007). *Electromagnetic Theory* . Wiley-IEEE Press.
9. Riley, KF, Hobson, MP, & Bence , SJ (2006). *Mathematical Methods for Physics and Engineering* . Cambridge University Press.
10. Landau, LD, & Lifshitz , EM (1984). *Electrodynamics of Continuous Media* (Vol. 8). Butterworth-Heinemann .