NAMANGAN DAVLAT PEDAGOGIKA INSTITUTI "TA'LIM VA TARAQQIYOT" ILMIY-USLUBIY JURNALI 2025-YIL 5-SON
13.00.00. – PEDAGOGIKA FANLARI JOURNAL.NAMSPI.UZ ISSN: 2992-9008; UDK: 37

APPLICATION OF INVERSION IN SOLVING STRUCTURAL PROBLEMS IN THE PROCESS OF TEACHING GEOMETRY

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Abstract: This article analyzes the role of the concept of inversion (rotation) in geometry teaching, its application to solving construction problems, and its methodological potential. Inversion is considered one of the important geometric transformations in geometry, revealing potential for simplifying complex geometric problems, optimizing constructions, and developing students' spatial imagination. The article examines analytical and geometric interpretations of inversion, as well as methods for visualizing it in digital educational environments (in GeoGebra, Desmos, and other programs). The study's results demonstrate that integrating inversion into the educational process promotes the development of students' creative thinking.

Keywords: *inversion, geometry, construction problems, spatial thinking, methodology, teaching technology, GeoGebra, visualization, learning process.*

ПРИМЕНЕНИЕ ИНВЕРСИИ ПРИ РЕШЕНИИ СТРУКТУРНЫХ ЗАДАЧ В ПРОЦЕССЕ ОБУЧЕНИЯ ГЕОМЕТРИИ.

Аннатация: В данной статье анализируется роль понятия инверсии (поворота) в процессе обучения геометрии, его применение при решении задач на построение и методические возможности. Инверсия рассматривается как одно из важных геометрических преобразований в геометрии, с её помощью раскрываются возможности упрощения сложных геометрических задач, оптимизации построений и развития пространственного воображения учащихся. В статье рассматриваются аналитические и геометрические интерпретации инверсии, а также способы её визуализации в цифровой образовательной среде (в программах GeoGebra, Desmos и других). Результаты исследования показывают, что внедрение инверсии в учебный процесс способствует развитию творческого мышления учащихся.

Ключевые слова: *инверсия, геометрия, задачи на построение, пространственное мышление, методика, технология обучения, GeoGebra, визуализация, процесс обучения.*

GEOMETRIYA FANINI O'QITISH JARAYONIDA INVERSIYANING YASASHGA DOIR MASALALAR YECHISHDAGI TATBIQLARI.

Annotatsiya: Ushbu maqolada geometriya fanini o'qitish jarayonida inversiya (aylantirish) tushunchasining o'rni, uning yasashga doir masalalarni yechishda tatbiqi va metodik imkoniyatlari tahlil qilingan. Inversiya geometriyada muhim o'rin tutuvchi geometrik o'zgarishlardan biri sifatida qaraladi va u yordamida murakkab geometrik masalalarni soddalashtirish, yasashlarni optimallashtirish hamda o'quvchilarning fazoviy tasavvurini rivojlantirish imkoniyatlari ochib berilgan. Maqolada inversiyaning analitik va geometrik talqinlari, shuningdek, raqamli ta'lim muhitida (GeoGebra, Desmos va boshqa dasturlar orqali) uni vizuallashtirish metodlari yoritiladi. Tadqiqot natijalari inversiyani o'qitish jarayoniga kiritish o'quvchilarning ijodiy fikrlashini rivojlantirishga xizmat qilishini ko'rsatadi.

Kalit so'zlar: *inversiya, geometriya, yasash masalalari, fazoviy tafakkur, metodika, o'qitish texnologiyasi, GeoGebra, vizuallashtirish, o'quv jarayoni.*

INTRODUCTION

The main goal of the modern education system is to develop students' independent thinking, analytical reasoning, and problem-solving skills. Geometry plays a special role in achieving this goal, fostering spatial imagination, logical analysis, and creative thinking.

In geometry, the concept of inversion (from the Latin inversio, meaning "reversal") is one of the classic geometric transformations based on the reflection of figures across a circle or sphere. Inversion can be used to simplify complex geometric figures, transform lines and circles into each other, and optimize geometric constructions.

From a pedagogical perspective, teaching inversion is important at advanced levels of geometry courses, especially when exploring topics in analytic and metric geometry in depth. It helps students:

understand the essence of geometric transformations;

combine visual and logical thinking;

develops the ability to analyze and transform new problems.

RESEARCH METHODOLOGY

Modern digital technologies, particularly interactive programs such as GeoGebra, Cabri 3D, and Desmos, modeling the inversion process, and visually explaining it to students enhance learning. Therefore, studying the use of inversion in teaching geometry is a methodologically relevant issue.

In this method, instead of directly determining the relationship between the target figure and the figures given in the problem, the relationship between the figures corresponding to their inverses is first found, and then the target figure is supplied. This is done in the following order:

In this case, the desired figure is considered to be found and is approximately drawn.

The goal is to find a point as the center of inversion such that, when the given and asked variables are inverted about a circle drawn with that point as the center, a simpler way of solving the problem is found, that is, the relationship between the figures corresponding to the inversion is simpler than the relationship between the given and asked variables in the problem.

When constructing an inversion circle that satisfies this condition, the given and sought values in the problem are replaced by inversions relative to this circle. By studying the relationships between the resulting inversion figures, we determine whether it is possible to construct a figure corresponding to the given one. This determines a method for solving an auxiliary problem that is simpler than the given one. This concludes the solution analysis phase.

RESEARCH AND RESULTS

By following the steps described in step 3 of the analysis (at the creation stage), a figure corresponding to the given one is created. Then, by performing an inverse substitution with respect to the selected circle, we find the desired figure.

Issue 1 Draw a circle passing through two given points and intersecting the given circle.

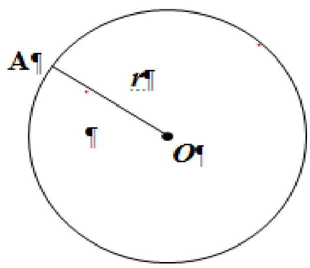

Figure 1

Analysis. The desired circle is the circle passing through points M and N in Figure 1 and intersecting the given circle A(O, R).A_xLet's assume it's a circle. Taking

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JOURNAL.NAMSPI.UZ
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one of the given points, say point M, as the center of inversion, we find that this centerWe draw an inversion circle u of arbitrary radius.

*she*point N relative to the circle, A and A_x circlesinversion replaced and compared themN'Let's form a point, a circle, and a line. By our assumption, since the desired circle passes through the given points and is tangent to the circle, the line also passes through the point and is tangent to the circle. Therefore, we find the line by solving an auxiliary problem: "Plot a path from a given point to a given circle." Then, by inverting the solution with respect to , we find the circle. Thus, the line is an auxiliary figure, since it can be constructed from the given points and transferred to the desired figure.$A'A_x'A_xAA_x'N'A'A_x'N'A'A_x'uA_xA_x'A_x$

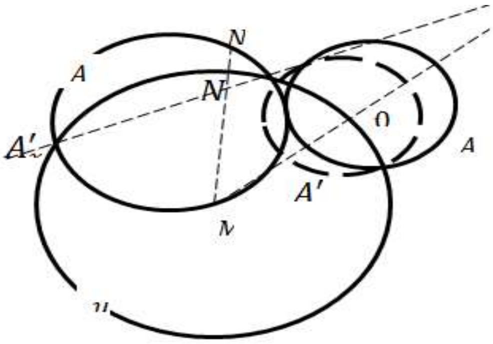

Figure 2

Manufacturing1. Taking one of the given points, for example, point , as the center of inversion, we draw from this center an inversion circle of arbitrary radius.Mu

2. We generate a point and a circle by inverting the given point and circle with respect to the inverse circle.$NAuN'A'$

3. We conduct experiments from a point to a circle (there are two experiments in total, only one experiment is shown in the figure).$N'A'A'_{x_1}A'_{x_2}$

4. Substituting the obtained attempts into the inverse rotation, we obtain the requested and rotations.$uA_{x_1}A_{x_2}$

<u>Issue 2</u>. *Draw a circle passing through two given points and intersecting a given line.*

The required circle is shown in Figure 2.A_{x_1}Let's assume that a circle passes through given points M and N and touches a given line l. Taking one of the given

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points, say point M, as the center of inversion, we draw an inversion circle of arbitrary radius from this center.u

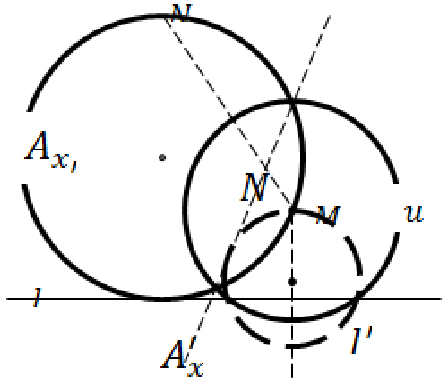

Figure 3

u We invert the figures relative to the circle and construct the corresponding line, point, and circle. According to our hypothesis, since the circle passes through the point and is tangent to the line, the line is also tangent to the circle passing through the point. Therefore, we can find the line; to do this, we make trials from the point to the circle and . We invert the resulting trials to form the desired circles and (one such circle is shown in the analysis figure).$A_{x_1}N, lA'_{x_1}N'l'A_{x_1}lA'_xN'l'A'_{x_1}N'l'A'_{x_1}A'_{x_2}A'_{x_1}A'_{x_2}$

<u>Issue 3</u>. *Draw a circle that intersects a given line at a given point on a given circle.*

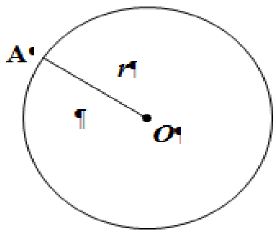

Figure 4

Analysis. The required circle is shown in Figure 3.ALet us assume that it consists of a circle and a line intersecting at point M.lA_{x_1}

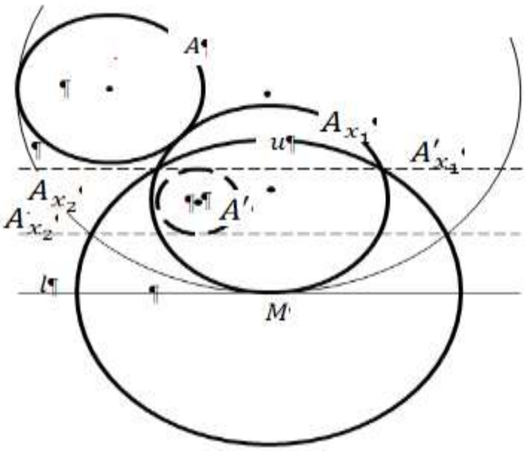

Figure 5

Point M is called the center of inversion, and its center is at point M.u Taking the circle as an inversion circle, we associate it with$l$If we replace a straight line and a circle with an inversion, then the inversion corresponding to a straight line will itself become a circle and a straight line will be formed.$AA_{x_1}lA'A'_{x_1}$

This study of the relationship between inversion-invariant figures shows that since a circle and a line intersect at the center of inversion, the inverted line corresponding to the circle is parallel to this line. And since the circles intersect, the inverted line corresponding to them also intersects the circle. This can be determined from the fact that the line intersects the circle and is parallel to the line. We find this by solving the auxiliary problem "Draw a line tangent to a given circle and parallel to a given line."$A_{x_1}lA_{x_1}A'_{x_1}lA'_{x_1}AA'_{x_1}A'A'_{x_1}A'lA'l$

We solve this problem, which is much simpler than the given one, by replacing the inversion with a solution that gives the desired circle.

Manufacturing1. Take a circle with its center at a given point and its radius as the inverse circle, and swap the line and the circle relative to it. The result is the line and the circle themselves.$MulAlA'$

2. Let's try to describe a circle and draw a parallel straight line.$lA'A'_{x_1}$ A'_{x_2}

3. If you replace the drawn attempts with inversion, then the drawn and rotated ones are formed.$A_{x_1}A_{x_2}$

Issue 4 Draw a circle that intersects the given circle at the given point and the given line.

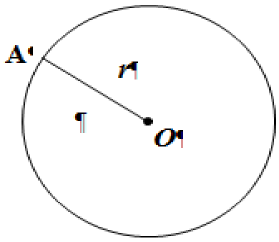

Figure 6

Analysis. Let the circle in the figure be the desired circle. Let it be tangent to the given circle at a point and to the given line. Let the circle drawn from the center with an arbitrary radius be the inversion circle. Replace the circle, the line, and the desired circle with it; this replacement yields a line, a circle, and the corresponding line.$A_{x_1}AMlMAlA_{x_1}A'l'A'_{x_1}$

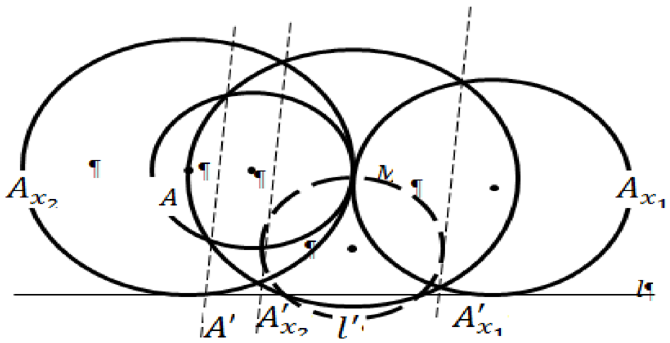

Figure 7

Now let's explore the relationships between these figures: since the circles intersect at the center, they are inverted, and the straight lines are parallel to each other.$AA_{x_1}MA'A'_{x_1}$

A_{x_1}Since the circle is tangent to the line, its inverse is proportional to it, and its images are proportional to each other. Therefore, a line inversely compatible with a given circle has the following property: the line is parallel to the line and tangent to the circle. These two properties define the line and can also be used to transfer it to the given circle. Therefore, the line is an auxiliary figure, and the auxiliary problem is found by solving the auxiliary problem of its parallelism to the given line and drawing a circle. Solving this auxiliary problem yields two trials, and . Replacing these trials with respect to the given circle by the inverse, we obtain the desired circles and .$lA'_{x_1}l'A'_{x_1}A'_{x_1}A'l'A'_{x_1}A'_{x_1}A'_{x_1}A'l'A'_{x_1}$ $A'_{x_2}A_{x_1}$ A_{x_2}

Issue 5 Draw a circle passing through the given point and tangent to the two given lines.

Analysis. Suppose the circle under consideration passes through a given point M in Figure 5 and touches lines a and b. Let us invert lines a and b, as well as the circle, relative to a circle drawn from the center M with an arbitrary radius. Let lines a and b touch circles a' and b' in the corresponding direction.$A_{x_1} A_{x_1}$ A_{x_1} A'_{x_1}

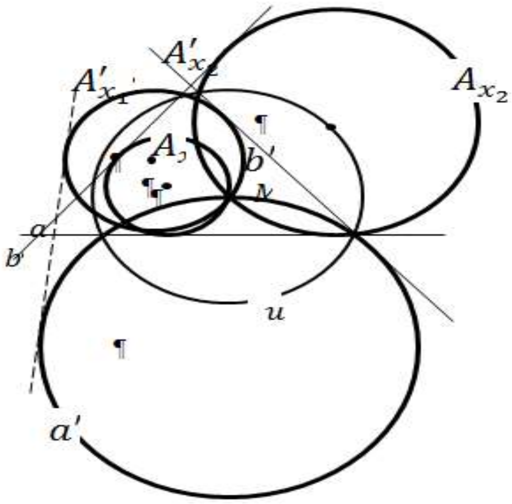

Figure 8

Therefore, to solve the problem, first the lines a and b are inverted with respect to an arbitrary circle u with center at point M and the resulting circles a' and b' are subject to generalizations, and then the resulting generalizations are inverted with respect to circle u and the desired circles are formed.

CONCLUSION

Using the inversion method in teaching geometry is an effective means of developing students' spatial thinking, deepening analytical thinking, and fostering creativity. Research findings. When using inversion, geometric construction problems are simplified and the visual accuracy of the solution is increased;

Teaching inversion using digital technology increases students' interest in science;

Lessons organized using interactive methods contribute to the development of students' logical and creative competencies. Thus, the application of inversion to solving construction problems is considered a methodological innovation in the teaching of geometry, and its integration with the digital educational environment increases the effectiveness of the learning process and allows for the development of education based on innovative approaches.

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