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MATEMATIKADAN OLIMPIADA MASALALARINI YECHISHDA MATEMATIK ANALIZ METODLARIDAN FOYDALANISH

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Annotatsiya: *Ushbu maqolada matematikadan olimpiada masalalarini yechishda matematik analizning ba'zi metodlaridan foydalanish usullari ko'rsatib berilgan.*

Kalit so'zlar: *olimpiada masalalari, mantiqiy fikrlash, matematik analiz, differensial hisob, eng katta qiymat, eng kichik qiymat.*

ИСПОЛЬЗОВАНИЕ МЕТОДОВ МАТЕМАТИЧЕСКОГО АНАЛИЗА ПРИ РЕШЕНИИ ОЛИМПИЙСКИХ ЗАДАЧ ПО МАТЕМАТИКЕ

Аннотация: *В статье рассматриваются использование некоторых методов решения олимпиадных задач по основ математического анализа. Некоторые задачи - подготовительного характера. Работа ориентируется на начинающего математика-студента первых курсов и ученика старших классов средней школы.*

Ключевые слова: *олимпийские проблемы, логическое мышление, математический анализ, дифференциальное исчисление, наибольшее значения, наименьшее значения.*

USE OF METHODS OF MATHEMATICAL ANALYSIS IN SOLVING OLYMPIC PROBLEMS IN MATH

Annotation: *The article shows the use of some methods for solving Olympiad problems on the basics of mathematical analysis. Some of the tasks are preparatory. The work is focused on a beginning mathematician-student of the first year and a student of the upper secondary school.*

Key words: *Olympic problems, logical thinking, mathematical analysis, differential calculus, largest value, smallest value.*

KIRISH

Ta'lim muassasalarida matematika o'qitishning asosiy vazifasi o'quvchi yoshlarni vatanga sadoqat, yuksak ahloq, ma'naviy boylikka ega bo'lish va mehnatga vijdonan munosabatda bo‘lish ruhida tarbiyalashga qaratilgan. Ta’limning insonparvar bo‘lishiga erishish, hozirgi zamon bozor iqtisodiyoti sharoitlarini hisobga olib har bir jamiyat a’zosini mehnat faoliyati va kundalik hayoti uchun zarur matematik bilim, ko‘nikma va malakani berishdan iborat.

So‘nggi yillarda xalqaro olimpiadalarda bilimli, iqtidorli yoshlarimiz muvaffaqiyati yildan-yilga yaxshilanib bormoqda. Biz yoshlarimizni bundan ham yuqori natijalarga erishib, davlatimiz obro‘–e’tiborini yanada oshiradi degan umiddamiz. Bizning maqsadimiz yosh avlodni layoqati, qobiliyati, iqtidorini aniqlash, ochish va ularning rivojlanishi uchun imkoniyat yaratishdan iboratdir.

Zero, olimpiada masalalari elementar va oliy matematikaning eng jozibador masalalar to‘plamidir. Olimpiada masalalari o‘quvchini chuqur fikrlashga, o‘z ustida ishlab, iqtidorini-malakasini takomillashtirishga, boy ijodiy tafakkurga ega bo‘lishga, qat’iyatli inson bo‘lishga va qaror qabul qila olishga o‘rgatadi.

Ma’lumki, olimpiada masalalari o‘quvchilarni mantiqiy fikrlash bilan birga o‘z xulosalarini asoslashga undaydi. Masalalarni yechish davomida o‘quvchilar nazariy bilimlarni takrorlaydi va uni amaliy jihatdan qo‘llash ko‘nikmasiga ega bo‘ladi. Matematikadan olimpiada masalalarini yechishda matematik analiz metodlaridan foydalanishni o‘rganish va ulardan foydalanish yo‘llarini topish o‘quvchilarni matematikaga bo‘lgan qiziqishlarini orttiradi. Ushbu ishda differensial hisobning tatbiqlarini keltirishni maqsad qilib oldik.

ADABIYOTLAR TAHLILI VA METODLAR

Mazkur maqola mazmuni va mavzusiga oid dastlabki ma’lumotlarni T.Azlarov, H.Mansurov [3] va Mal Coad va boshqalarning [4] adabiyotlaridan topish mumkin. Matematika analizning metodlaridan foydalanib yechish mumkin bo‘lgan matematik olimpiada misol va masalalarni M.A.Mirzaahmedov, Sh.N.Ismailov [1] va X.Norjigitov, J.A.Bahramovlarning [2] adabiyotlaridan topish mumkin.

Maqola matematikadan olimpiada masalalarini yechishda matematik analiz metodlarining roli va o‘rnini ko‘rsatib berishga bag‘ishlangan. Bunda asosiy diqqat differensial hisobning asosiy teoremalariga qaratilgan.

NATIJALAR VA MUHOKAMA

Differensial hisobning asosiy teoremalari yordamida yechiladigan masalalar. Biz bu bandda funksiya hosilasini bir nechta masalalarga tatbiqini keltiramiz.

1-masala. Agar $a \geq 0$, $b \geq 0$, $p \geq 1$ bo'lsa, $\left(\frac{a+b}{2}\right)^p \leq \frac{a^p + b^p}{2}$ bo'lishini isbotlang.

Isboti. Agar $a = 0$ yoki $b = 0$ bo'lsa, tengsizlikni tenglik sharti bajariladi. Shuning uchun $a \geq b > 0$ faqat holatni qaraymiz va

$$\left(\frac{a+b}{2}\right)^p \leq \frac{a^p + b^p}{2} \Leftrightarrow \left(\frac{\frac{a}{b}+1}{2}\right)^p \leq \frac{\left(\frac{a}{b}\right)^p + 1}{2}$$

ekanini inobatga olamiz. Ushbu funksiyani tuzamiz:

$$f(x) = \frac{x^p + 1}{2} - \left(\frac{x+1}{2}\right)^p, \quad x \geq 1, \; p \geq 1.$$

U holda $f(0) = 0$, $f'(x) = \frac{p}{2}\left(x^{p-1} - \left(\frac{x+1}{2}\right)^p\right) \geq 0$ bo'ladi. $f'(x) \geq 0$ bo'lgani uchun $f(x)$ funksiya o'suvchi. Shuning uchun $f(x) \geq f(0)$.

Bundan $\frac{x^p + 1}{2} \geq \left(\frac{x+1}{2}\right)^p$ bo'ladi. $x = \frac{a}{b}$ qilib tanlasak

$$\left(\frac{a+b}{2}\right)^p \leq \frac{a^p + b^p}{2}$$

tengsizlikka ega bo'lamiz.

2-masala. c ning hech bir qiymatida $x(x^2 - 1)(x^2 - 10) = c$ tenglama beshta butun yechimga ega bo'la olmasligini isbotlang.

Isboti. Ushbu funksiyani tuzamiz: $f(x) = x(x^2 - 1)(x^2 - 10)$.

Bu funksiya butun sonlar o'qida aniqlangan va $f'(x) = 5x^4 - 3x^2 + 10$.

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1). $f'(x)=0$ tenglamani yechamiz. Bundan

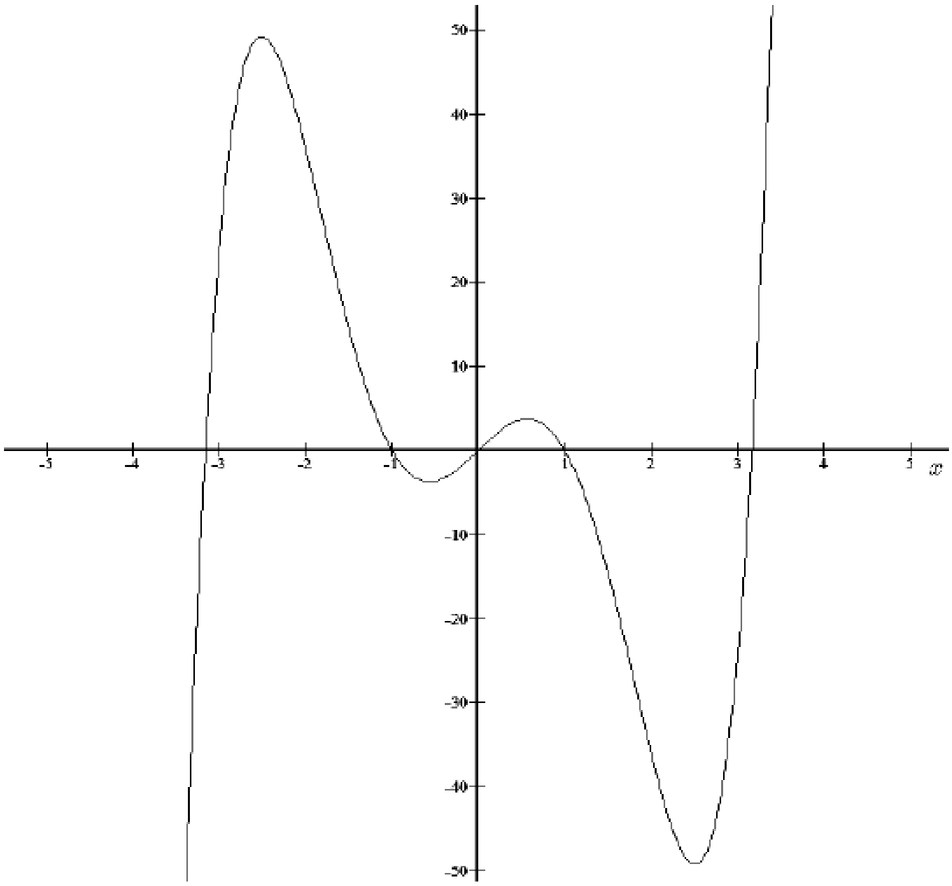
$$x_1, x_4 = \pm\sqrt{\frac{33+\sqrt{889}}{10}} \text{ va } x_2, x_3 = \pm\sqrt{\frac{33-\sqrt{889}}{10}}$$

kelib chiqadi.

2). $f(x)=0$ tenglamani yechamiz.

$$x(x^2-1)(x^2-10)=0 \Rightarrow x=0,\ x=\pm1 \text{ va } x=\pm\sqrt{10}\ .$$

3). Funksiya grafigini yasaymiz. (1-rasm)



1-rasm

Funksiyaning monotonlik oraliqlari beshta:

1) $(-\infty; x_1]$, 2) $[x_1; x_2]$, 3) $[x_2; x_3]$, 4) $[x_3; x_4]$, 5) $[x_4; +\infty)$.

Shuning uchun $y=c$ to'g'ri chiziq $f(x)$ funksiyani ko'pi bilan beshta nuqtada kesishi mumkin. $[x_1; x_2]$ $([x_1; x_2] \subset (-1;1))$ monotonlik oralig'ida yagona $x=0$ butun son bor.

Demak, $c=0$ bo‘lgandagi $f(x)=c$ tenglama ko‘pi bilan beshta butun yechimga ega bo‘lishi mumkin.

$f(x)=0$ tenglama esa $x=0$ va $x=\pm1$ butun yechimlarga ega. Bu holat $f(x)=c$ tenglama beshta butun yechimga ega emasligini bildiradi.

Funksiya hosilasini ba’zi murakkab masalalariga tadbiqlari. Ushbu bandda funksiya hosilasining ba’zi murakkab masalalar yechishga tatbiqlarini ko‘rib chiqamiz.

1-teorema. $ABCD$ to‘g‘ri to‘rtburchakda ixtiyoriy M nuqta olingan bo‘lib, $|AB|=a$, $|AD|=b$, $\lambda\ge1$ bo‘lsa, u holda

a) $\max\left\{|MA|^{\lambda}+|MB|^{\lambda}+|MC|^{\lambda}+|MD|^{\lambda}\right\}=a^{\lambda}+b^{\lambda}+\left(\sqrt{a^2+b^2}\right)^{\lambda}$;

b) $\min\left\{|MA|^{\lambda}+|MB|^{\lambda}+|MC|^{\lambda}+|MD|^{\lambda}\right\}=4\left(\dfrac{a^2+b^2}{4}\right)^{\lambda}$

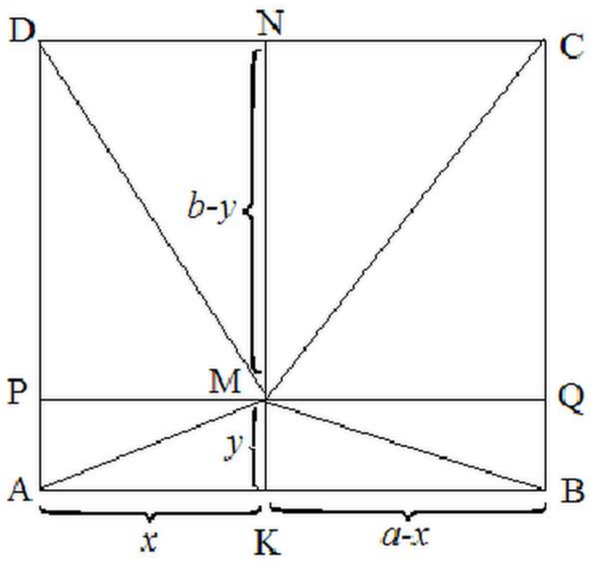
tengliklar o‘rinli bo‘ladi.

Isbot. M nuqtadan $KN\perp AB$, $PQ\perp AD$ kesmalarni o‘tkazamiz (1-chizma). Aytaylik, $|AK|=x$, $|MK|=y$ bo‘lsin. M nuqta to‘rtburchakda bo‘lganligi uchun $0\le x\le a$, $0\le y\le b$ bo‘ladi. Pifagor teoremasiga ko‘ra:

$$|MA|^{\lambda}+|MB|^{\lambda}+|MC|^{\lambda}+|MD|^{\lambda}=\left(x^2+y^2\right)^{\frac{\lambda}{2}}+\left((x-a)^2+y^2\right)^{\frac{\lambda}{2}}+$$
$$+\left((x-a)^2+(y-b)^2\right)^{\frac{\lambda}{2}}+\left(x^2+(y-b)^2\right)^{\frac{\lambda}{2}}.$$

Ushbu belgilashni kiritib olib, quyidagi ikki holni ko‘rib chiqamiz.

$$P(x,y)=\left(x^2+y^2\right)^{\frac{\lambda}{2}}+\left((x-a)^2+y^2\right)^{\frac{\lambda}{2}}+\left((x-a)^2+(y-b)^2\right)^{\frac{\lambda}{2}}+\left(x^2+(y-b)^2\right)^{\frac{\lambda}{2}}$$



2-rasm

1-hol. $y=0$ yoki $y=b$ bo‘lsin. Bu holda

$$P(x,0)=P(x,b)=x^{\lambda}+(a-x)^{\lambda}+\left((x-a)^2+b^2\right)^{\frac{\lambda}{2}}+\left(x^2+b^2\right)^{\frac{\lambda}{2}}$$

bo‘lganligi sababli faqat $P(x,0)$ ni o‘rganish yetarli. $P(x,0)$ funksiyani $(0,\ a)$ oraliqda x bo‘yicha birinchi va ikkinchi tartibli hosilalarini hisoblaymiz:

$$P'(x,0)=\lambda\left(x^{\lambda-1}-(a-x)^{\lambda-1}+(x-a)\left((x-a)^2+b^2\right)^{\frac{\lambda}{2}-1}+x\left(x^2+b^2\right)^{\frac{\lambda}{2}-1}\right)$$

$$P''(x,0)=\lambda\left((\lambda-1)\left(x^{\lambda-2}-(a-x)^{\lambda-2}\right)+\left((x-a)^2+b^2\right)^{\frac{\lambda}{2}-2}\left((\lambda-1)(x-a)^2+b^2\right)\right)+$$

$$+\lambda\left(x^2+b^2\right)^{\frac{\lambda}{2}-2}\left((\lambda-1)x^2+b^2\right).$$

$P''(x,0)\geq 0$ bo‘lgani uchun $P'(x,0)$ funksiya $(0,a)$ oraliqda o‘suvchi. Shuning uchun $P'(x,0)=0$ tenglama $(0,a)$ oraliqda ko‘pi bilan bitta yechimga ega bo‘lishi mumkin. $P'\left(\frac{a}{2},0\right)=0$ bo‘lgani sababli, yagona yechim $x=\frac{a}{2}$ dan iborat bo‘ladi. Demak, $P(x,0)$ funksiya $\left[0,\frac{a}{2}\right]$ kesmada kamayuvchi, $\left[\frac{a}{2},a\right]$ kesmada esa o‘suvchi bo‘ladi. Bunga asosan quyidagi tengliklarga ega bo‘lamiz:

$$\max_{0\le x\le a} P(x,0)=P(0,0)=P(a,0)=a^{\lambda}+b^{\lambda}+\left(\sqrt{a^2+b^2}\right)^{\lambda},$$

$$\min_{0\le x\le a} P(x,0)=P\left(\frac{a}{2},0\right)=2\left(\frac{a}{2}\right)^{\lambda}+2\left(\left(\frac{a}{2}\right)^2+b^2\right)^{\frac{\lambda}{2}}.$$

2-hol. $0<y<b$ bo'lsin. Bu holda y o'zgaruvchini tayinlab, $P(x,y)$ funksiyani x bo'yicha birinchi va ikkinchi tartibli hosilalarini hisoblaymiz:

$$P'(x,y)=\lambda x\left(x^2+y^2\right)^{\frac{\lambda}{2}-1}+\lambda(x-a)\left((x-a)^2+y^2\right)^{\frac{\lambda}{2}-1}+$$

$$+\lambda(x-a)\left((x-a)^2+(y-b)^2\right)^{\frac{\lambda}{2}-1}+\lambda x\left(x^2+(y-b)^2\right)^{\frac{\lambda}{2}-1},$$

$$P''(x,y)=\lambda\left(x^2+y^2\right)^{\frac{\lambda}{2}-2}\left(x^2(\lambda-1)+y^2\right)+$$

$$+\lambda\left((x-a)^2+y^2\right)^{\frac{\lambda}{2}-2}\left((x-a)^2(\lambda-1)+y^2\right)+$$

$$+\lambda\left((x-a)^2+(y-b)^2\right)^{\frac{\lambda}{2}-2}\left((\lambda-1)(x-a)^2+(y-b)^2\right)+$$

$$+\lambda\left(x^2+(y-b)^2\right)^{\frac{\lambda}{2}-2}\left(x^2(\lambda-1)+(y-b)^2\right).$$

Yuqorida qilingan ishlarni takrorlab, ushbu

$$\max_{0\le x\le a} P(x,y)=P(0,y)=P(a,y)=y^{\lambda}+\left(a^2+y^2\right)^{\frac{\lambda}{2}}+\left(a^2+(y-b)^2\right)^{\frac{\lambda}{2}}+(b-y)^{\lambda}$$

$$\min_{0\le x\le a} P(x,y)=P\left(\frac{a}{2},y\right)=2\left(\left(\frac{a}{2}\right)^2+y^2\right)^{\lambda}+2\left(\left(\frac{a}{2}\right)^2+(y-b)^2\right)^{\frac{\lambda}{2}}$$

tengliklarni hosil qilamiz.

Endi ushbu

$$f(y)=y^{\lambda}+\left(a^2+y^2\right)^{\frac{\lambda}{2}}+\left(a^2+(y-b)^2\right)^{\frac{\lambda}{2}}+(b-y)^{\lambda},$$

$$g(y)=2\left(\left(\frac{a}{2}\right)^2+y^2\right)^{\frac{\lambda}{2}}+2\left(\left(\frac{a}{2}\right)^2+(y-b)^2\right)^{\frac{\lambda}{2}}$$

yordamchi funksiyalarni kiritib olamiz. Bu funksiyalar uchun $\max\limits_{0\le x\le b} f(y)$ va $\min\limits_{0\le x\le b} g(y)$ larni hisoblaymiz. Shu maqsadda, $(0,b)$ oraliqda, $f(y)$ va $g(y)$ funksiyalarning birinchi va ikkinchi tartibli hosilalarini hisoblaymiz:

$$f'(y)=\lambda y^{\lambda-1}+\lambda y\left(a^2+y^2\right)^{\frac{\lambda}{2}-1}+\lambda(y-b)\left(a^2+(y-b)^2\right)^{\frac{\lambda}{2}-1}-\lambda(b-y)^{\lambda-1},$$

$$f''(y)=\lambda(\lambda-1)y^{\lambda-2}+\lambda\left(a^2+y^2\right)^{\frac{\lambda}{2}-2}\left(a^2+(\lambda-1)y^2\right)+$$

$$+\lambda\left(a^2+(y-b)^2\right)^{\frac{\lambda}{2}-2}\left(a^2+(\lambda-1)(y-b)^2\right)+\lambda(\lambda-1)(b-y)^{\lambda-2},$$

$$g'(y)=2\lambda y\left(\left(\frac{a}{2}\right)^2+y^2\right)^{\frac{\lambda}{2}-1}+2\lambda(y-b)\left(\left(\frac{a}{2}\right)^2+(y-b)^2\right)^{\frac{\lambda}{2}-1},$$

$$g''(y)=2\lambda\left(\left(\frac{a}{2}\right)^2+y^2\right)^{\frac{\lambda}{2}-2}\left(\left(\frac{a}{2}\right)^2+(\lambda-1)y^2\right)+$$

$$+2\lambda\left(\left(\frac{a}{2}\right)^2+(y-b)^2\right)^{\frac{\lambda}{2}-2}\left(\left(\frac{a}{2}\right)^2+(\lambda-1)(y-b)^2\right).$$

Bu ifodalardan $0\le y\le b$ bo‘lganida $f''(y)\ge 0$, $g''(y)\ge 0$ va $f'\left(\frac{b}{2}\right)=0$, $g'\left(\frac{b}{2}\right)=0$ kelib chiqadi. Demak, $f(y)$ va $g(y)$ funksiyalar funksiyalar $y=\frac{b}{2}$ nuqtada o‘zining eng kichik qiymatlariga erishadi, kesmaning chetki nuqtalarida esa eng katta qiymatlarga erishadi, xususan

$$\max_{0\le x\le b} f(y)=f(0)=f(b)=a^\lambda+b^\lambda+\left(\sqrt{a^2+b^2}\right)^\lambda,$$

$$\min_{0\le x\le b} g(y)=g\left(\frac{b}{2}\right)=4\left(\frac{a^2+b^2}{4}\right)^{\frac{\lambda}{2}}$$

bo'ladi. Birinchi va ikkinchi hollarni hisobga olsak, a) va b) tengliklar kelib chiqadi. Teorema isbotlandi.

Natija. Agar $\lambda = 1$ bo'lsa, ushbu

$$\max\{|MA| + |MB| + |MC| + |MD|\} = a + b + \sqrt{a^2 + b^2},$$

$$\min\{|MA| + |MB| + |MC| + |MD|\} = 2\sqrt{a^2 + b^2}$$

Tengliklar, $\lambda = 2$ bo'lsa, quyidagi

$$\max\{|MA|^2 + |MB|^2 + |MC|^2 + |MD|^2\} = 2(a^2 + b^2),$$

$$\min\{|MA|^2 + |MB|^2 + |MC|^2 + |MD|^2\} = a^2 + b^2$$

tengliklar o'rinli bo'ladi.

1-masala. Yuzasi S bo'lgan $ABCD$ – qavariq to'rtburchakning AB, BC, CD, DA tomonlarida mos ravishda M, N, P, Q nuqtalar shunday olinganki, bunda

$$\frac{|AM|}{|MB|} = \frac{|BN|}{|NC|} = \frac{|CP|}{|PD|} = \frac{|DQ|}{|QA|}.$$

$MNPQ$ to'rtburchak yuzasining eng kichik qiymatini toping.

Yechish. Aytaylik, $\frac{|AM|}{|MB|} = \lambda$ bo'lsin. U holda

$$|AB| = |AM| + |MB| = (1 + \lambda)|MB|, \ |BC| = |BN| + |NC| = \frac{\lambda + 1}{\lambda}|BN|,$$

$$|AD| = |AQ| + |QD| = \frac{\lambda + 1}{\lambda}|QD|, \ |CD| = |CP| + |pD| = \frac{\lambda + 1}{\lambda}|PD|$$

bo'ladi. Bularga asosan quyidagilarga ega bo'lamiz (2-chizma):

$$S_{PQD} = \frac{\lambda}{(1 + \lambda)^2} \cdot S_{ACD}, \quad S_{MBN} = \frac{\lambda}{(1 + \lambda)^2} \cdot S_{ABC}.$$

Bundan ushbu tenglik kelib chiqadi: $S_{PQD} + S_{MBN} = \frac{\lambda}{(1 + \lambda)^2} \cdot S$.

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Xuddi shuningdek, $S_{QAM}+S_{NCP}=\frac{\lambda}{(1+\lambda)^2}\cdot S$ tenglikka ega bo'lamiz.

Demak,

$$S_{MNPQ}=S_{ABCD}-\left(S_{QAM}+S_{NCP}+S_{MBN}+S_{PDQ}\right)==S-\frac{2\lambda}{(1+\lambda)^2}\cdot S=\frac{\lambda^2+1}{(1+\lambda)^2}\cdot S$$

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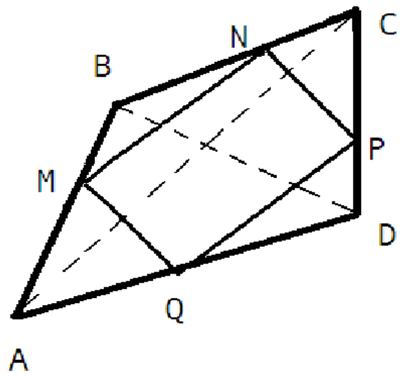

3-rasm

Oxirgi ifodaning eng kichik qiymatini topish maqsadida ushbu $f(\lambda)=\frac{\lambda^2+1}{(1+\lambda)^2}$, $\lambda>0$ funksiyani kiritib olamiz va uning eng kichik qiymatini topamiz. Buning uchun avvalo uning hosilasini hisoblaymiz:

$$f'(\lambda)=\frac{2\lambda\cdot(\lambda^2+1)-(\lambda^2+1)\cdot 2(\lambda+1)}{(1+\lambda)^4}=\frac{2(\lambda-1)}{(1+\lambda)^3}.$$

Bu ifodaga binoan $f'(\lambda)=0$ tenglamaning yagona yechimi $\lambda=1$ bo'lib, $(0;1)$ oraliqda $f'(\lambda)<0$ va $(1;+\infty)$ oraliqda $f'(\lambda)>0$ bo'ladi. Bu esa $f(\lambda)$ funksiyaning eng kichik qiymati $f(1)=\frac{1}{2}$ ga teng ekanini bildiradi. Demak, $MNPQ$ to'rtburchak yuzasining eng kichik qiymati $\frac{1}{2}S$ ga teng bo'lib, bu qiymat yuqoridagi nisbat 1 ga teng bo'lganda, ya'ni M, N, P, Q nuqtalar to'rtburchak tomonlarining o'rtalarida bo'lganida erishiladi.

2-masala. Agar a, b, c – musbat sonlar bo'lsa, u holda ixtiyoriy λ, μ, $(\lambda \geq \mu \geq 0)$ sonlari uchun ushbu

$$\frac{a^\lambda}{b^\lambda + c^\lambda} + \frac{b^\lambda}{a^\lambda + c^\lambda} + \frac{c^\lambda}{a^\lambda + b^\lambda} \geq \frac{a^\mu}{b^\mu + c^\mu} + \frac{b^\mu}{a^\mu + c^\mu} + \frac{c^\mu}{a^\mu + b^\mu} \qquad (1)$$

tengsizlik o'rinli bo'ladi.

Isbot. Quyidagi funksiyani qaraymiz:

$$f(\lambda) = \frac{p^\lambda}{q^\lambda + 1} + \frac{q^\lambda}{p^\lambda + 1} + \frac{1}{p^\lambda + q^\lambda}, \quad \lambda \geq 0.$$

Bunda p, q $(p \geq q \geq 1)$ – o'zgarmas sonlar. Bu funksiyaning hosilasini hisoblab, $p \geq q \geq 1$ bo'lgani uchun $f'(\lambda) \geq 0$ bo'lishini, ya'ni $f(\lambda)$ funksiya o'suvchi bo'lishini topamiz. Demak, har qanday λ, μ, $(\lambda \geq \mu \geq 0)$ – nomanfiy sonlar uchun $f(\lambda) \geq f(\mu)$, ya'ni

$$\frac{p^\lambda}{q^\lambda + 1} + \frac{q^\lambda}{p^\lambda + 1} + \frac{1}{p^\lambda + q^\lambda} \geq \frac{p^\mu}{q^\mu + 1} + \frac{q^\mu}{p^\mu + 1} + \frac{1}{p^\mu + q^\mu} \qquad (2)$$

tengsizlik o'rinli. Umumiylikka zid ish qilmagan holda $a \geq b \geq c$ deb olib, $p = \frac{a}{c}$, $q = \frac{b}{c}$ deb tanlasak, u holda (2) tengsizlik quyidagi ko'rinishga keladi:

$$\frac{\left(\frac{a}{c}\right)^\lambda}{\left(\frac{b}{c}\right)^\lambda + 1} + \frac{\left(\frac{b}{c}\right)^\lambda}{\left(\frac{a}{c}\right)^\lambda + 1} + \frac{1}{\left(\frac{a}{c}\right)^\lambda + \left(\frac{b}{c}\right)^\lambda} \geq \frac{\left(\frac{a}{c}\right)^\mu}{\left(\frac{b}{c}\right)^\mu + 1} + \frac{\left(\frac{b}{c}\right)^\mu}{\left(\frac{a}{c}\right)^\mu + 1} + \frac{1}{\left(\frac{a}{c}\right)^\mu + \left(\frac{b}{c}\right)^\mu}.$$

Bu tengsizlikning chap va o'ng tomonlarida shakl almashtirishlar bajarib, (1) tengsizlikni hosil qilamiz.

Endi (1) tengsizlikni ba'zi xususiy hollarni keltirib o'tamiz.

1. Agar $\lambda \geq 0$, $\mu = 0$ bo'lsa, ushbu tengsizlik hosil bo'ladi:

$$\frac{a^\lambda}{b^\lambda + c^\lambda} + \frac{b^\lambda}{a^\lambda + c^\lambda} + \frac{c^\lambda}{a^\lambda + b^\lambda} \geq \frac{3}{2}.$$

2. Agar $\lambda = 2$, $\mu = 1$ bo‘lsa, ushbu tengsizlik hosil bo‘ladi:

$$\frac{a^2}{b^2+c^2}+\frac{b^2}{a^2+c^2}+\frac{c^2}{a^2+b^2}\geq\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}$$

XULOSA

Ushbu ishda matematikadan olimpiada masalalarini yechishda matematik analiz metodlaridan foydalanishni o‘rganish va ularga doir turli masalalarning yechish uslubiyati ko‘rib chiqildi. Olimpiada masalalarini yechishda matematik analiz foydalanish usullari o‘rganildi va ularni olimpiada masalalarini yechishga tatbiq qilish uslubiyati ko‘rsatib berildi.

Matematikadan olimpiada masalalarini yechishda matematik analiz metodlaridan foydalanishni o‘rganish va ulardan foydalanish yo‘llarini topish o‘quvchilarni matematikaga bo‘lgan qiziqishlarini orttiradi, matematik analiz fanining o‘quv materialini chuqurroq va kengroq o‘rganish imkoniyatini yaratadi. Ushbu ish o‘quvchilarning bilimlarini va matematikaga bo‘lgan qiziqishlarini oshiradi hamda o‘quvchilarni matematikadan olimpidaga tayyorlashda to‘garak rahbarlariga bu ishning katta yordami tegadi, degan umiddamiz.

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